Optimization of Acoustic Black Holes for Vibration Reduction

30 August 2021

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GRADUATE PROGRAM
IN ACOUSTICS

Primary references

- A. Pelat, F. Gautier, S. C. Conlon, and F. Semperlotti, "The acoustic black hole: A review of theory and applications," *J. Sound Vib.* 476, 115316 (2020)
  - DOI: 10.1016/j.jsv.2020.115316

- C. Zhao and M. G. Prasad, "Acoustic black holes in structural design for vibration and noise control," *Acoustics* 1(1), 220-251 (2019).
  - DOI: 10.3390/acoustics1010014

Structure of the Talk

- Background
- Review of ABH research
- ABH optimization study
- Concluding remarks

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- Adding (viscoelastic) damping to the structure often leads to a significant increase in weight

- Acoustic black holes (ABHs) promise the potential for significant vibration reduction without increased weight

Classical ABH theory

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- Seminal paper by Mikhail A. Mironov (1988)

Propagation of a flexural wave in a plate whose thickness decreases smoothly to zero in a finite interval

M. A. Mironov

N. N. Andreev Acoustics Institute, Academy of Sciences of the USSR

(Submitted October 14, 1987)

Akust. Zh. 34, 546–547 (May–June 1988)

If the thickness of a plate varies sufficiently smoothly, a flexural wave can propagate in it without reflection. The local wave number k varies, because it depends on the thickness of the plate h, and the wave amplitude A varies because of the conservation of energy flux transported by the wave:

$$k = \left(\frac{3\rho\omega^2}{E_{p\ell} h^2(x)}\right)^{1/4}, \qquad (1)$$

$$A = A_0 (h_0/h(x))^{3/4}. \qquad (2)$$

Here x is the coordinate along the plate, ρ is the material density of the plate, $E_{p\ell}$ is the Young's modulus of the plate, and ω is the angular frequency. The condition of sufficient smoothness of the variation of the plate thickness is stated as follows: The variation of the flexural wave number must be small over a distance of wavelength order:

$$\frac{dk}{dx}\frac{1}{k} \ll k,$$

or

$$\frac{1}{k^2}\frac{dk}{dx} \ll 1. \qquad (3)$$

Substituting Eq. (1) in (3), we obtain

$$\frac{1}{2}\left(\frac{E_{p\ell}}{3\rho\omega^2}\right)^{1/4}\frac{1}{h^{1/2}}\frac{dh}{dx} \ll 1. \qquad (4)$$

Condition (4) is satisfied, in particular, by a power-law thickness that essentially vanishes in a finite interval:

$$h(x) = \varepsilon x^n. \qquad (5)$$

Substituting Eq. (5) in (4), we have

$$n x^{n-1} \ll 2(3\rho\omega^2/E_{p\ell})^{1/4}\frac{1}{\varepsilon^{1/2}} x^{n/2}.$$

This condition is satisfied at sufficiently small x for n > 2 and at all distances x for n = 2 and $\varepsilon \ll (3\rho\omega^2/E_{p\ell})^{1/2}$. Let us consider the case n = 2. The flexural wave number tends to infinity as $x \to 0$:

$$k = \left(\frac{3\rho\omega^2}{E_{p\ell}}\right)^{1/4}\frac{1}{\varepsilon^{1/2}}\frac{1}{x}, \qquad (6)$$

and the phase and group velocities tend to zero:

$$c_p = \frac{\omega}{k} = \varepsilon^{1/2}\left(\frac{E_{p\ell}\omega^2}{3\rho}\right)^{1/4} x,$$

$$c_{gr} = \partial\omega/\partial k = 2\varepsilon^{1/2}\left(\frac{E_{p\ell}\omega^2}{3\rho}\right)^{1/4} x. \qquad (7)$$

The transit time of a wave packet with the carrier frequency ω from the coordinate x_0 to the coordinate x_1 (see Fig. 1) is

$$T = \int_{x_0}^{x_1}\frac{dx}{c_{gr}} = \frac{1}{2\varepsilon^{1/2}}\left(\frac{3\rho}{E_{p\ell}\omega^2}\right)^{1/4}\left|\ln\frac{x_1}{x_0}\right|. \qquad (8)$$

According to Eq. (8), the transit time tends to infinity as $x_1 \to 0$. This means that a wave originating in the thick part of the plate does not reach the tapered edge in any finite time and is therefore not reflected from it. The wave energy is absorbed in the plate for any indefinitely small absorption. Assuming that Young's modulus is complex, $E_{p\ell} \to E_{p\ell}(1 + i/Q)$ (Q is the quality factor of the material), from Eq. (6) we obtain an equation for the imaginary part of the local wave number:

$$\operatorname{Im} k = \frac{1}{4Q}\operatorname{Re} k = \frac{1}{4Q}\left(\frac{3\rho\omega^2}{E_{p\ell}}\right)^{1/4}\frac{1}{\varepsilon^{1/2}}\frac{1}{x}.$$

As x tends to zero, Im k tends to infinity. Moreover, the integrated space rate of attenuation in the interval (x_0, x_1)

$$\int_{x_0}^{x_1}\operatorname{Im} k\, dx = \frac{1}{4Q\varepsilon^{1/2}}\left(\frac{3\rho\omega^2}{E_{p\ell}}\right)^{1/4}\left|\ln\left(\frac{x_1}{x_0}\right)\right| \qquad (9)$$

also tends to infinity as x_1 approaches zero.

Thus, a parabolically tapered plate has the property that it totally absorbs an incident flexural wave. This effect has analogs in other wave processes. It has been shown[1] that an internal wave in a horizontally smoothly inhomogeneous stratified fluid is not reflected from the cutoff plane of the wave. The problem of the wave function of a particle in a potential field $U(r) = -\gamma/r^n$ is discussed in a quantum mechanics textbook,[2] in which it is shown that discrete eigenvalues are nonexistent for n > 2 or for n = 2 and sufficiently large γ, and the particle hits the center of the field. In wave language, this means that reflection from the center does not take place. Pekeris[3] noted the absence of reflected waves in a model layered-inhomogeneous medium with a sound-velocity profile that decays linearly to zero with increasing depth.

The main difficulty in actually creating a nonreflecting edge of a plate lies in the precise shaping of the part of the plate near the edge. Since it is impossible to reduce the plate thickness to zero according to a parabolic law, a real plate is cut off at some finite thickness h_1 (see Fig. 1).

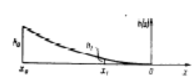
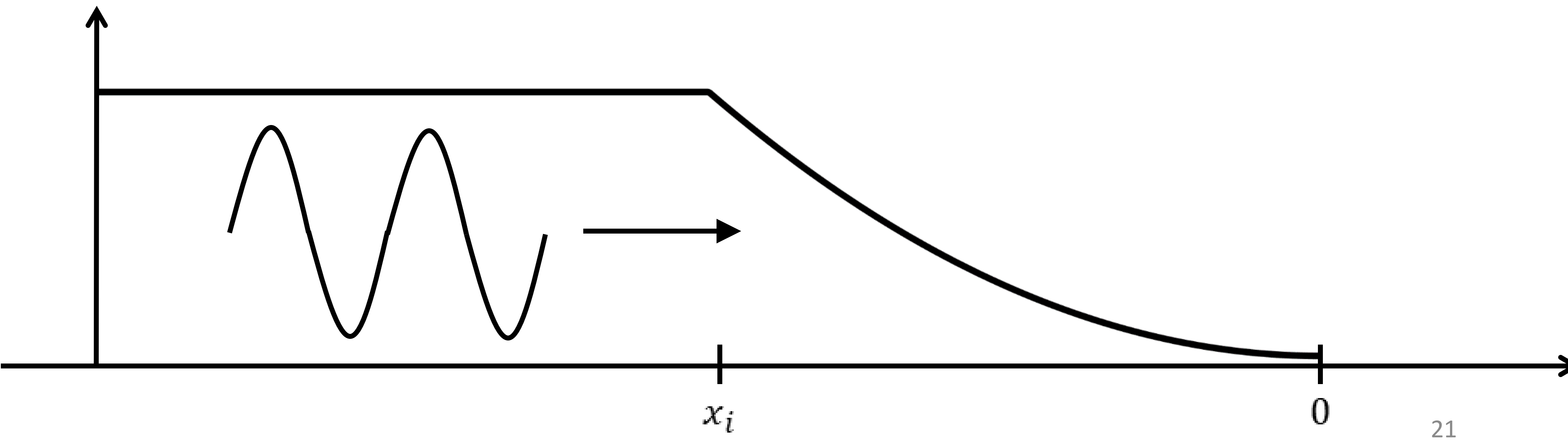


FIG. 1. Plate with a parabolically tapered edge; x = 0 is the coordinate of the edge of the ideal plate; $x = x_0$ is the coordinate of the initial cross section of thickness h_0; $x = x_1$ is the running coordinate where the thickness of the cross section is h_1, or the coordinate of the edge of the real plate, for which the parabolic variation of the thickness holds only down to h_1, at which point the plate is truncated.

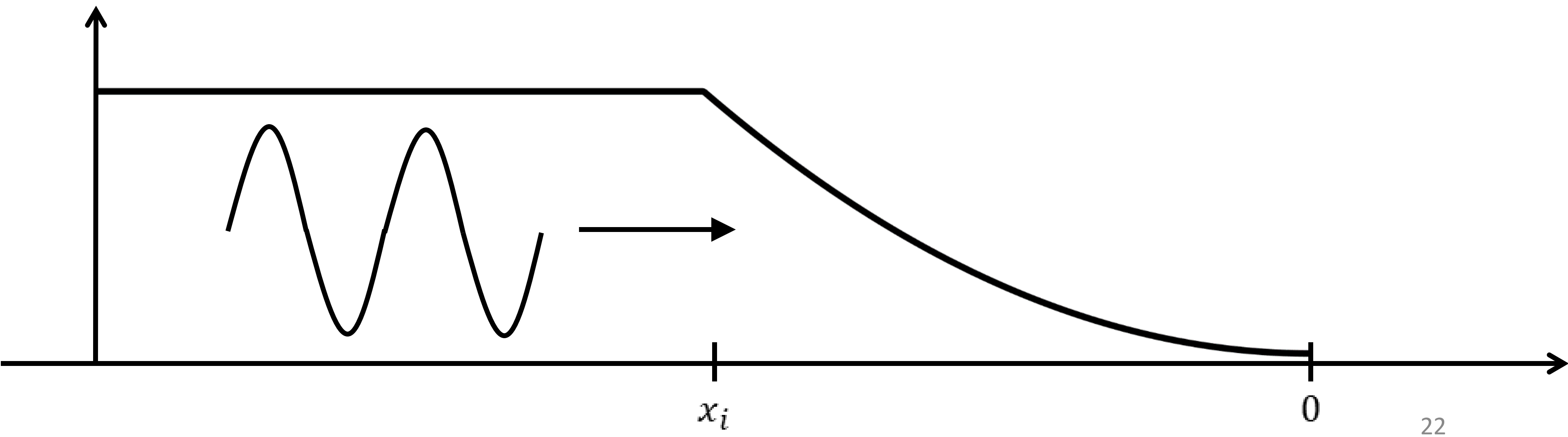
Classical ABH theory



Classical ABH theory

- Transit time to origin

$$\tau = \int_0^{x_i} \frac{1}{c_g(x)} \mathrm{d}x \propto \int_0^{x_i} \frac{1}{\sqrt{h(x)}} \mathrm{d}x$$



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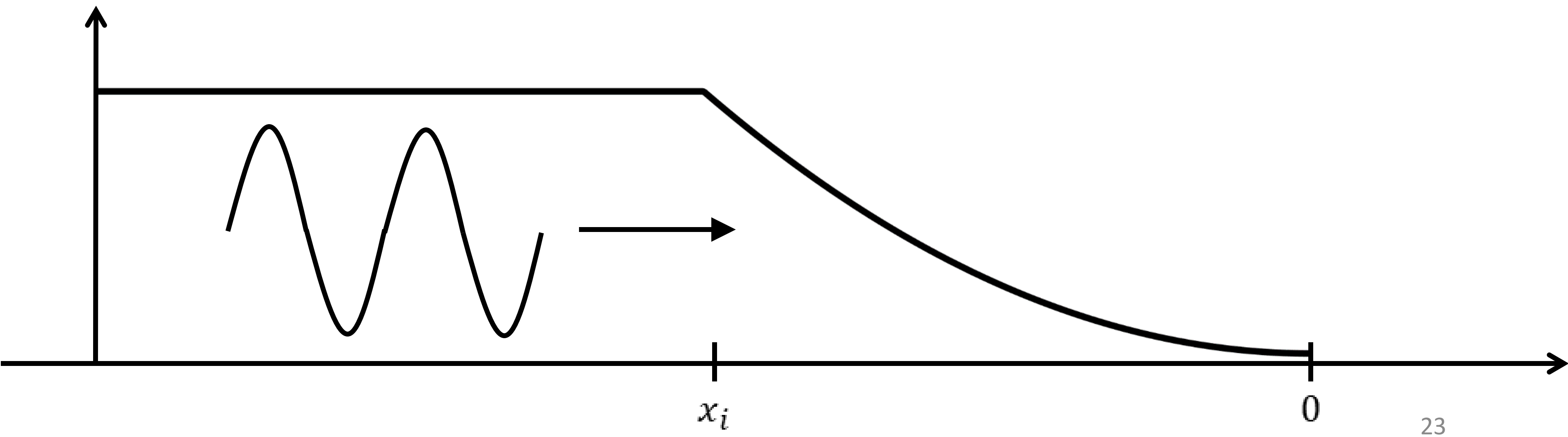
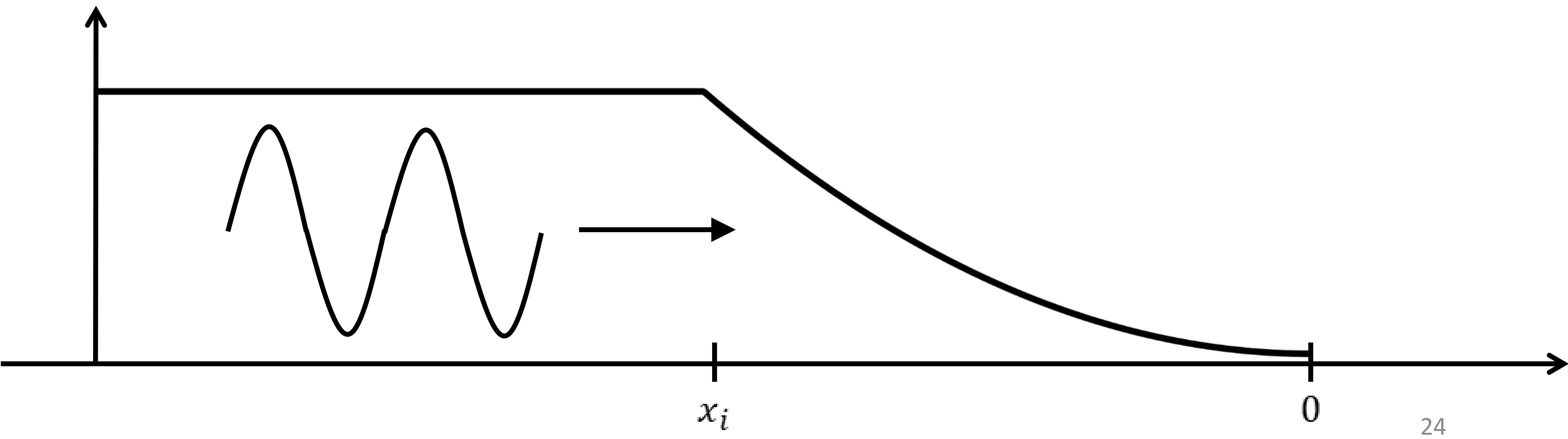
- If $h(x) = \varepsilon x^2$, then $\tau \to \infty$



Classical ABH theory

- Reflection

$$|R| \approx e^{-\omega\eta\boldsymbol{\tau}}$$

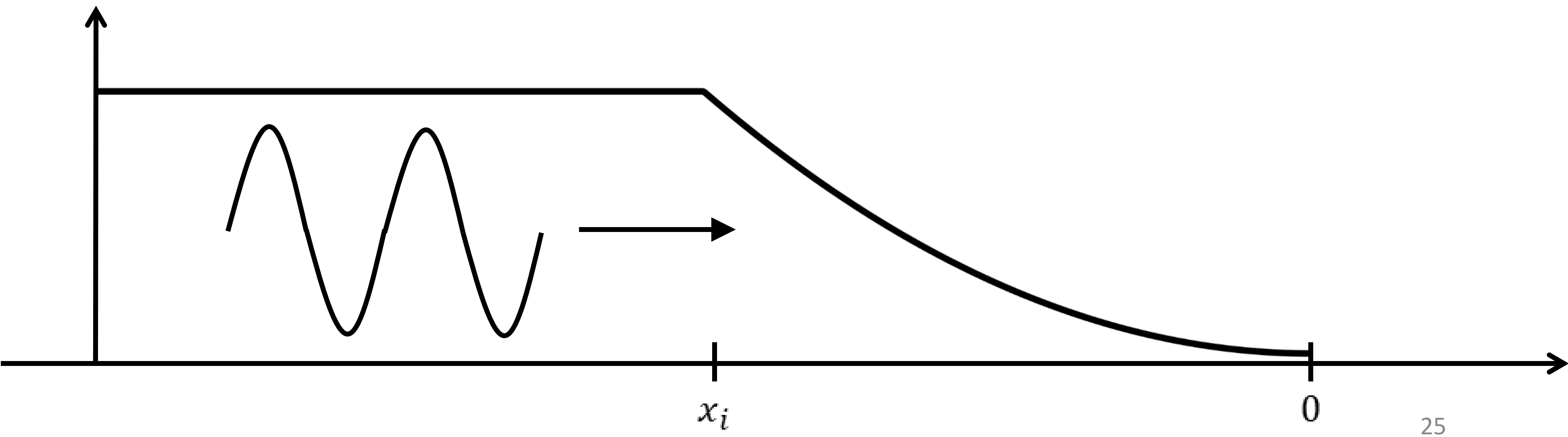


Classical ABH theory

- Reflection

$$|R| \approx e^{-\omega\eta\tau}$$

- If $h(x) = \varepsilon x^2 + h_0 > 0$, then $|R| > 0$



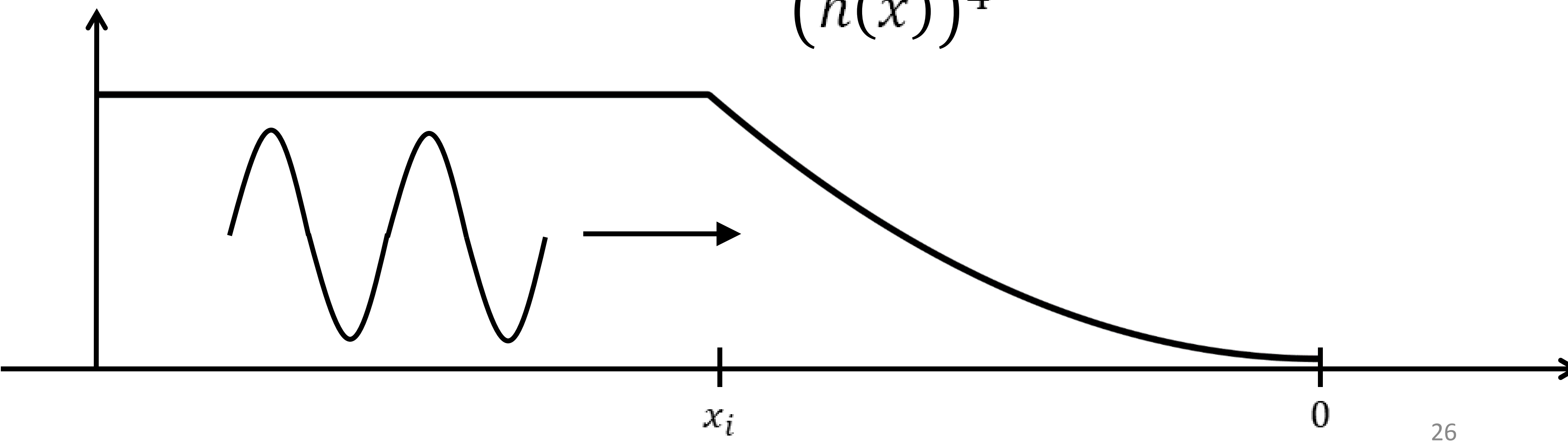
Classical ABH theory

- Wavelength

$$\lambda(x) \propto \sqrt{h(x)}$$

- Amplitude

$$A(x) \propto \frac{1}{\left(h(x)\right)^{\frac{3}{4}}}$$



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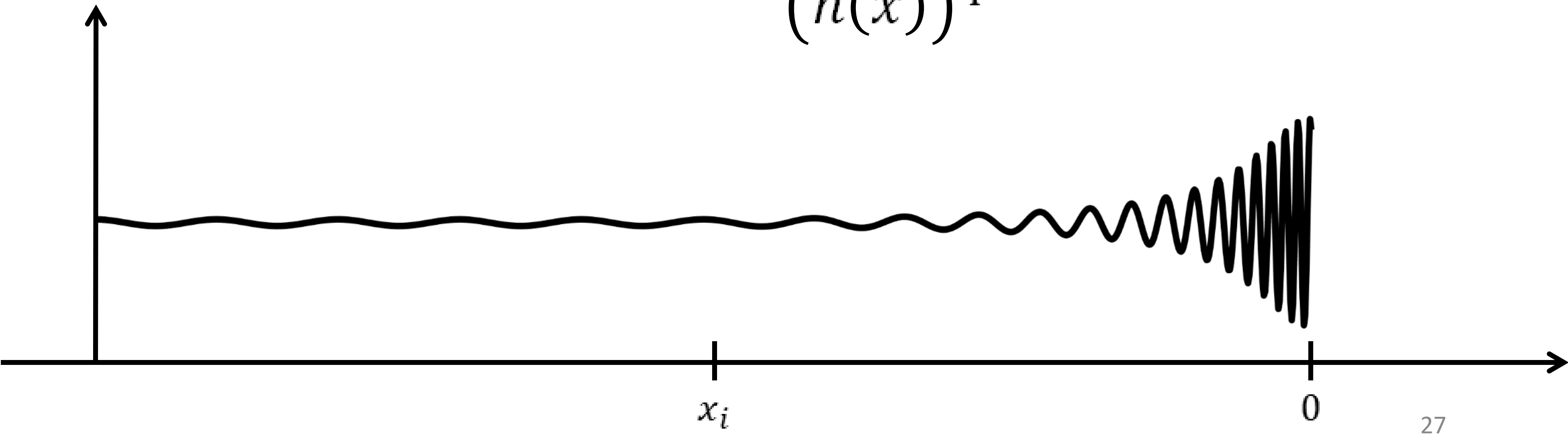
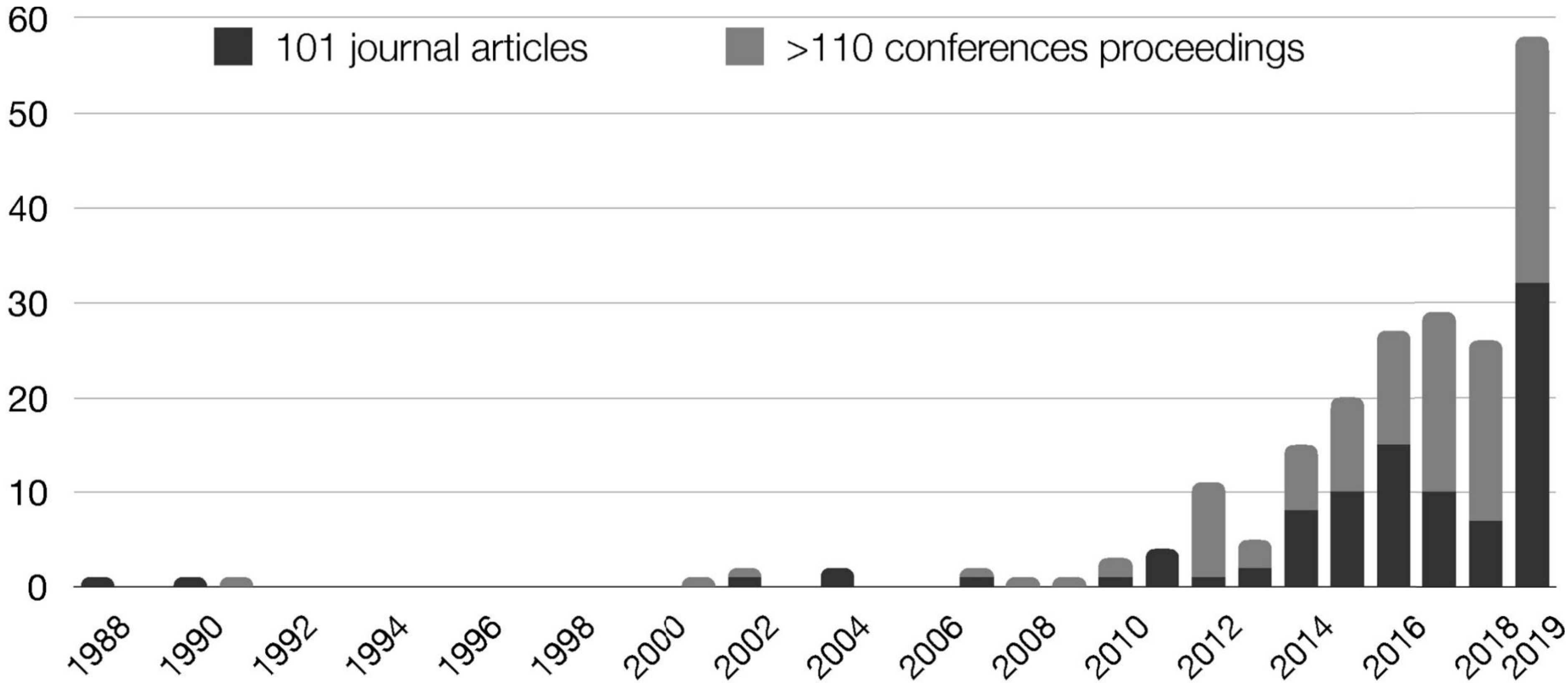
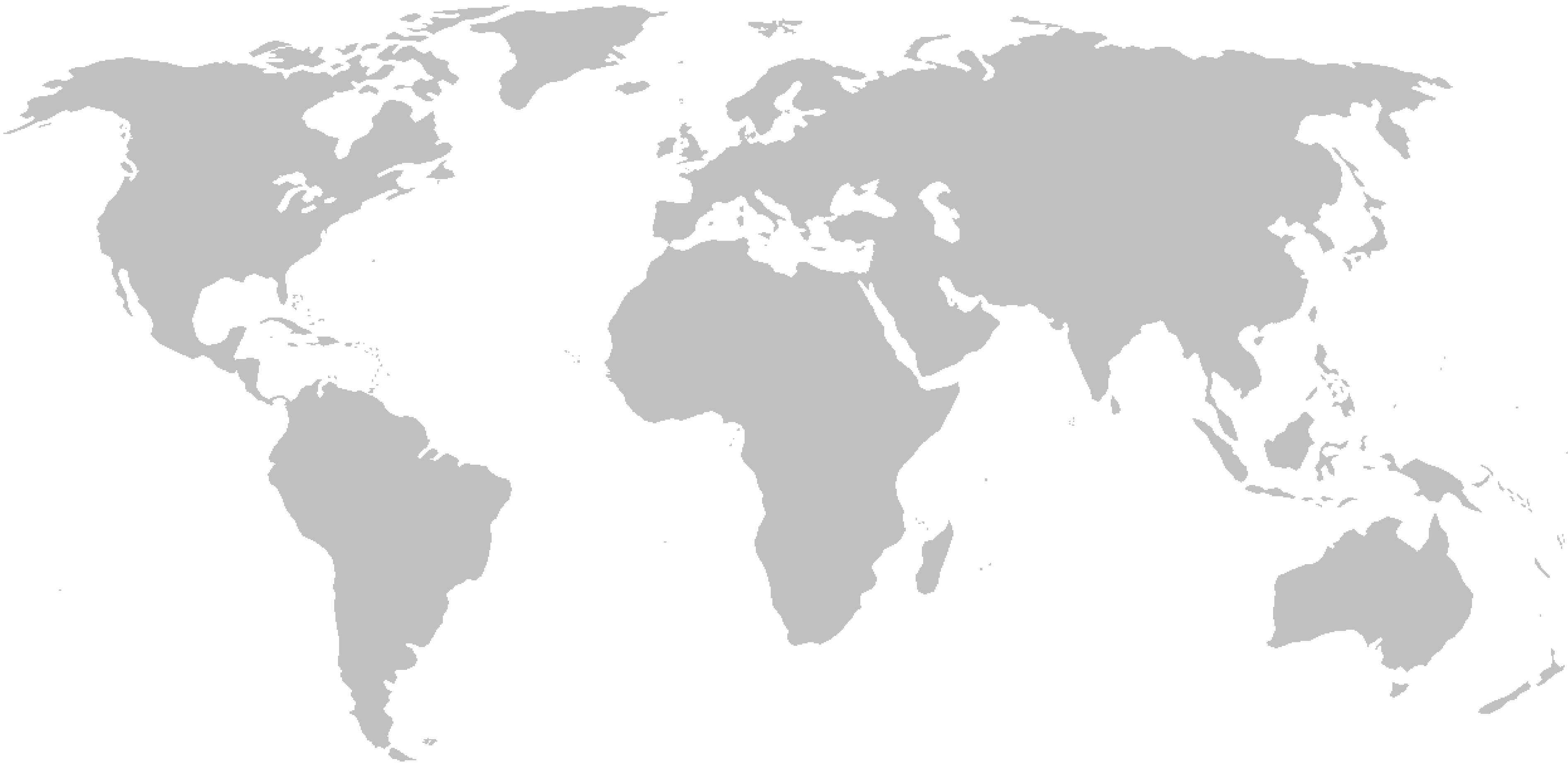


Publication history



J. Sound Vib. 476, 115316 (2020)

Research hubs



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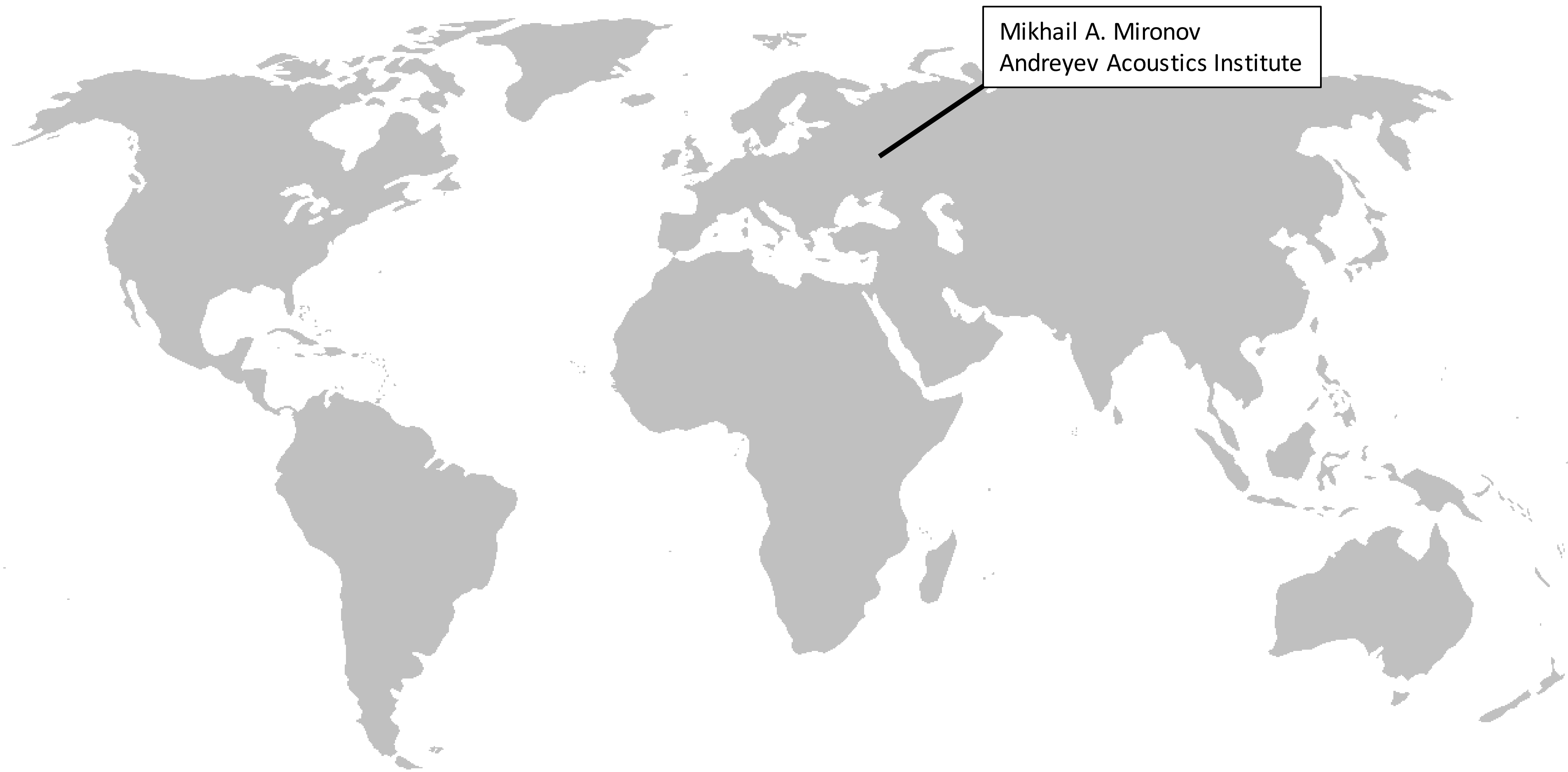

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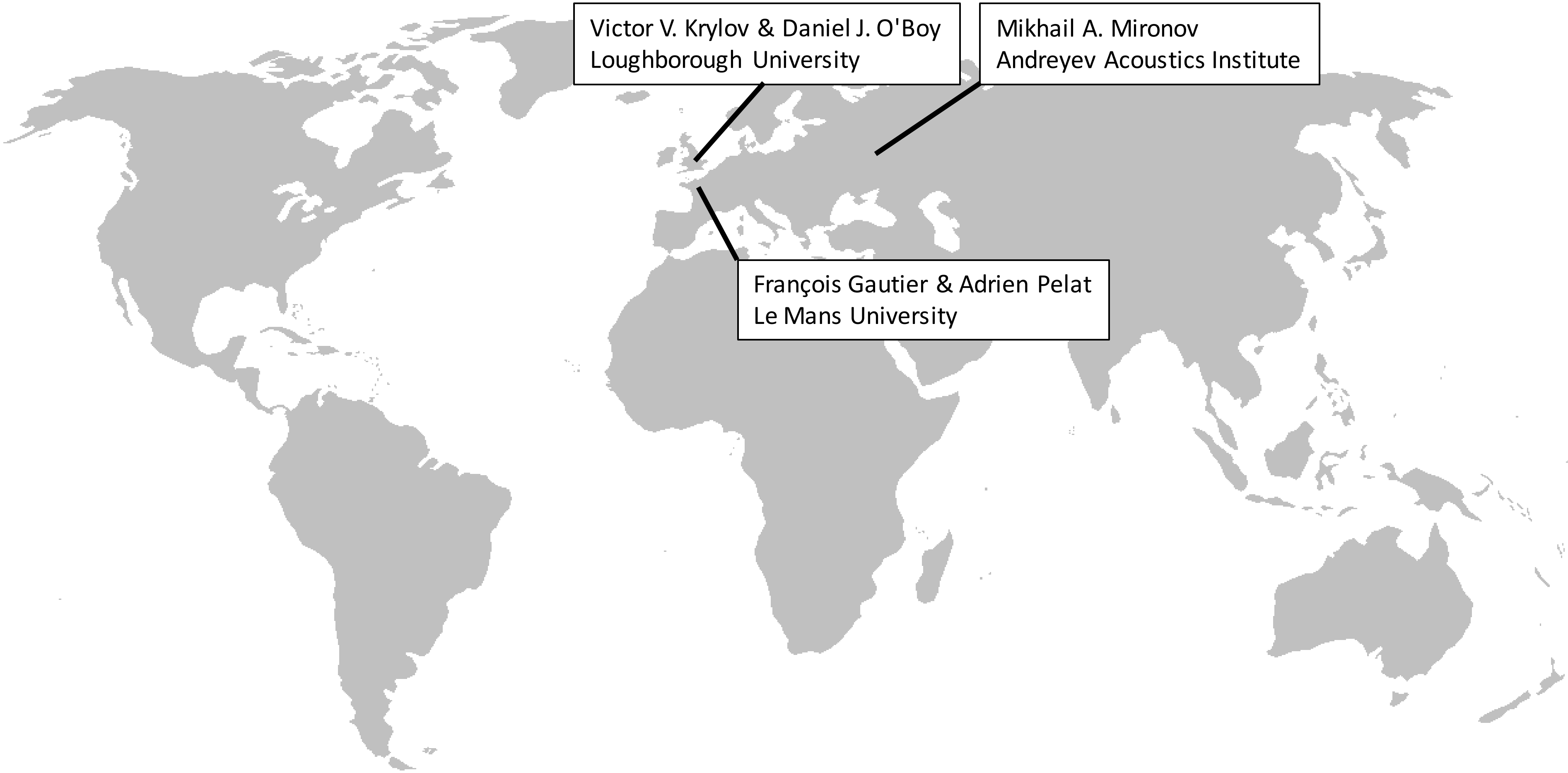

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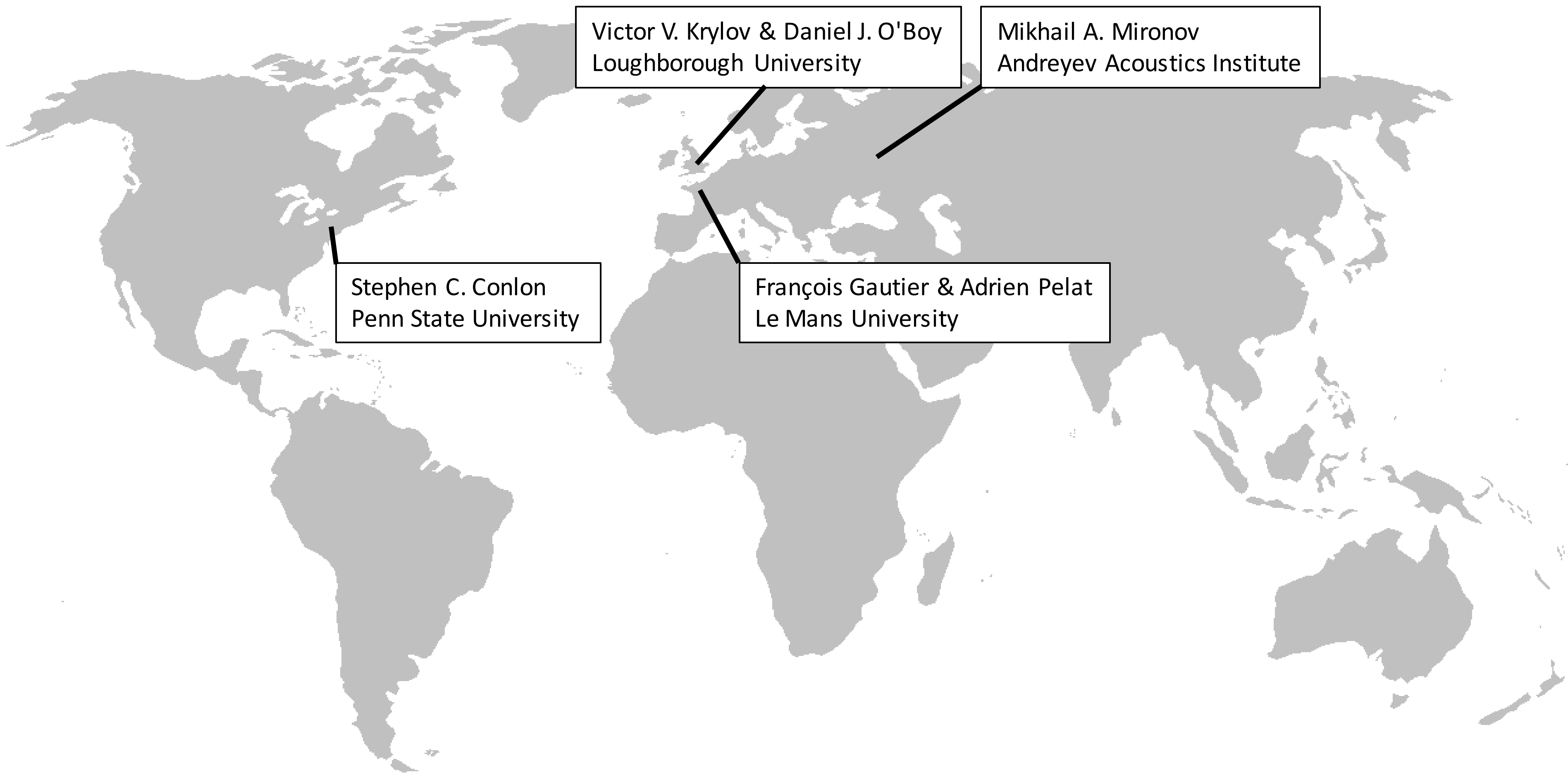

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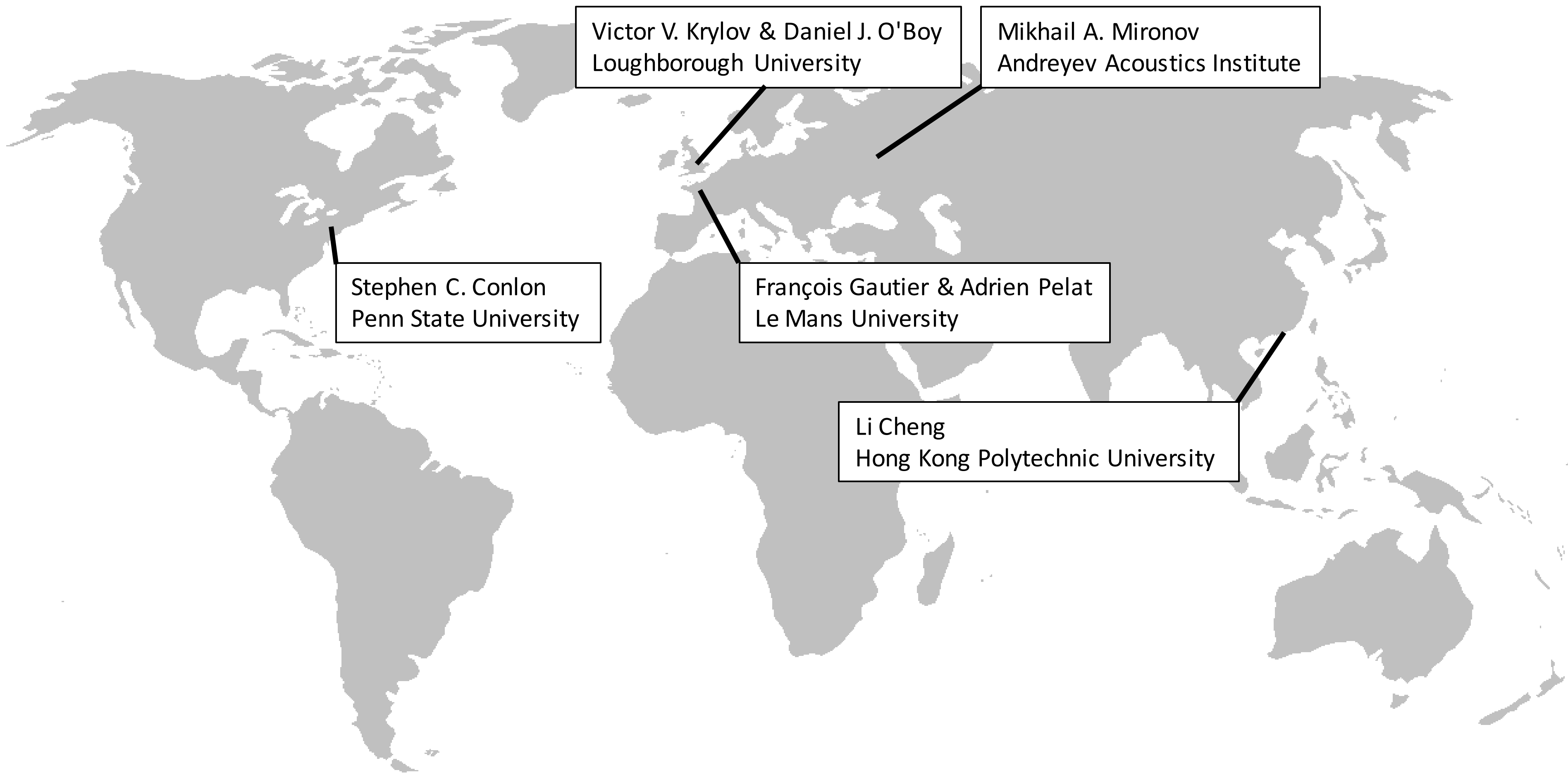

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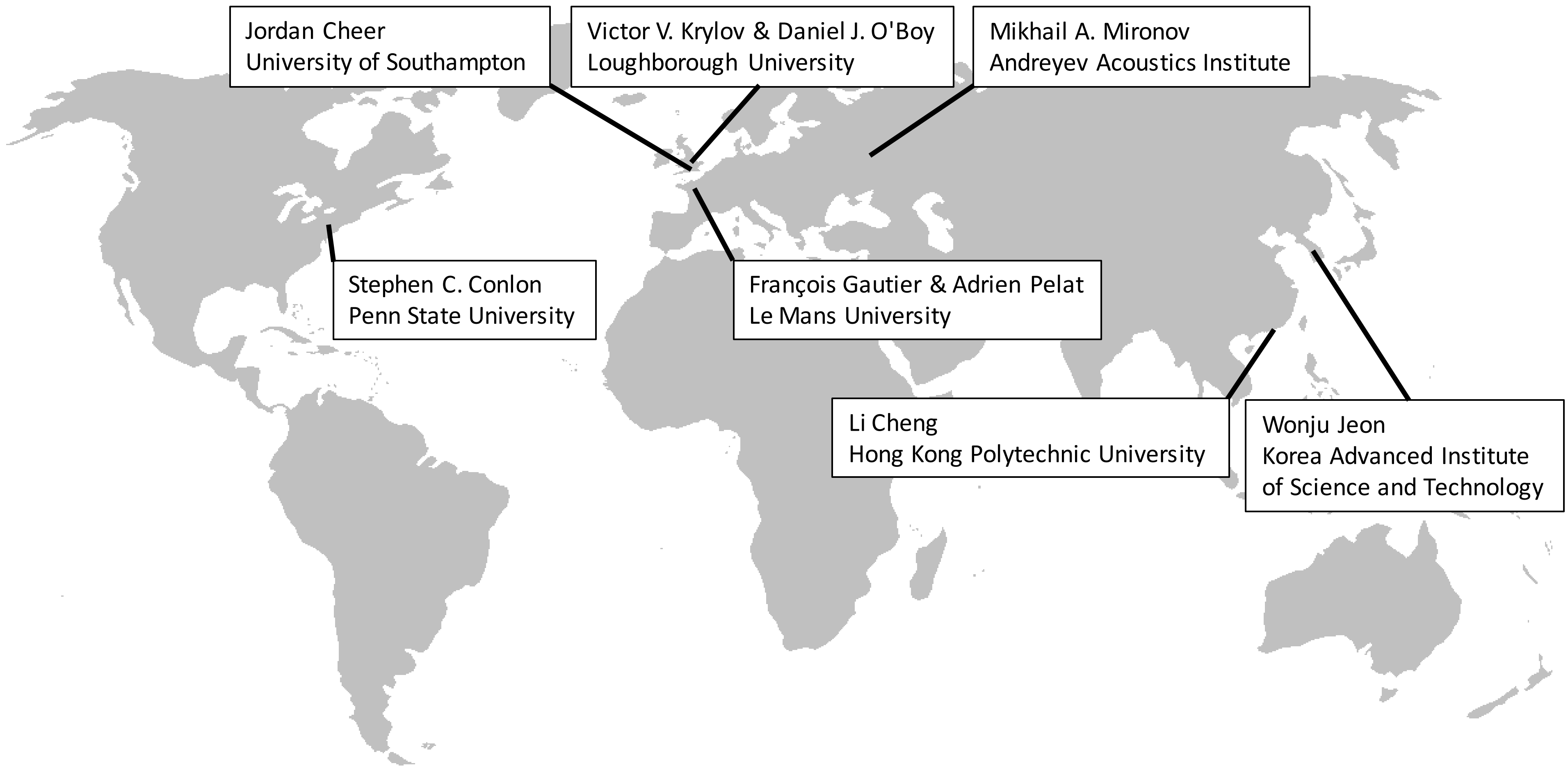
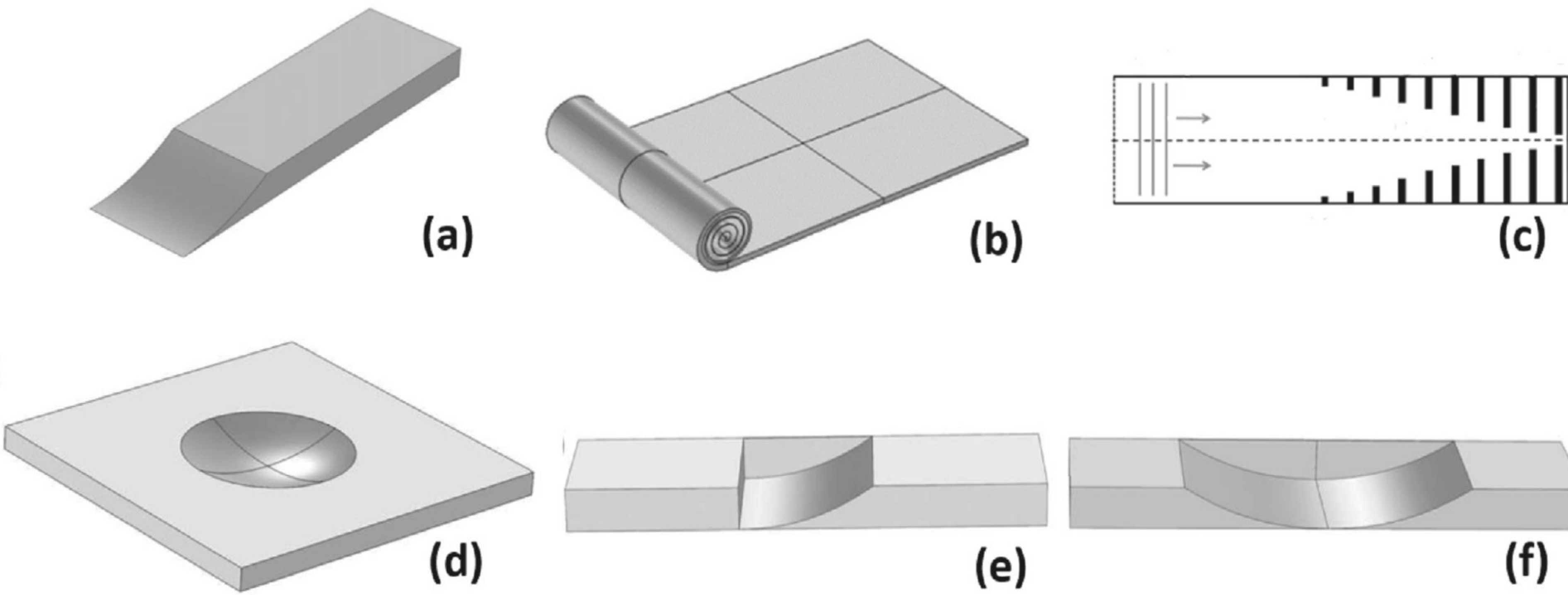


ABH variations



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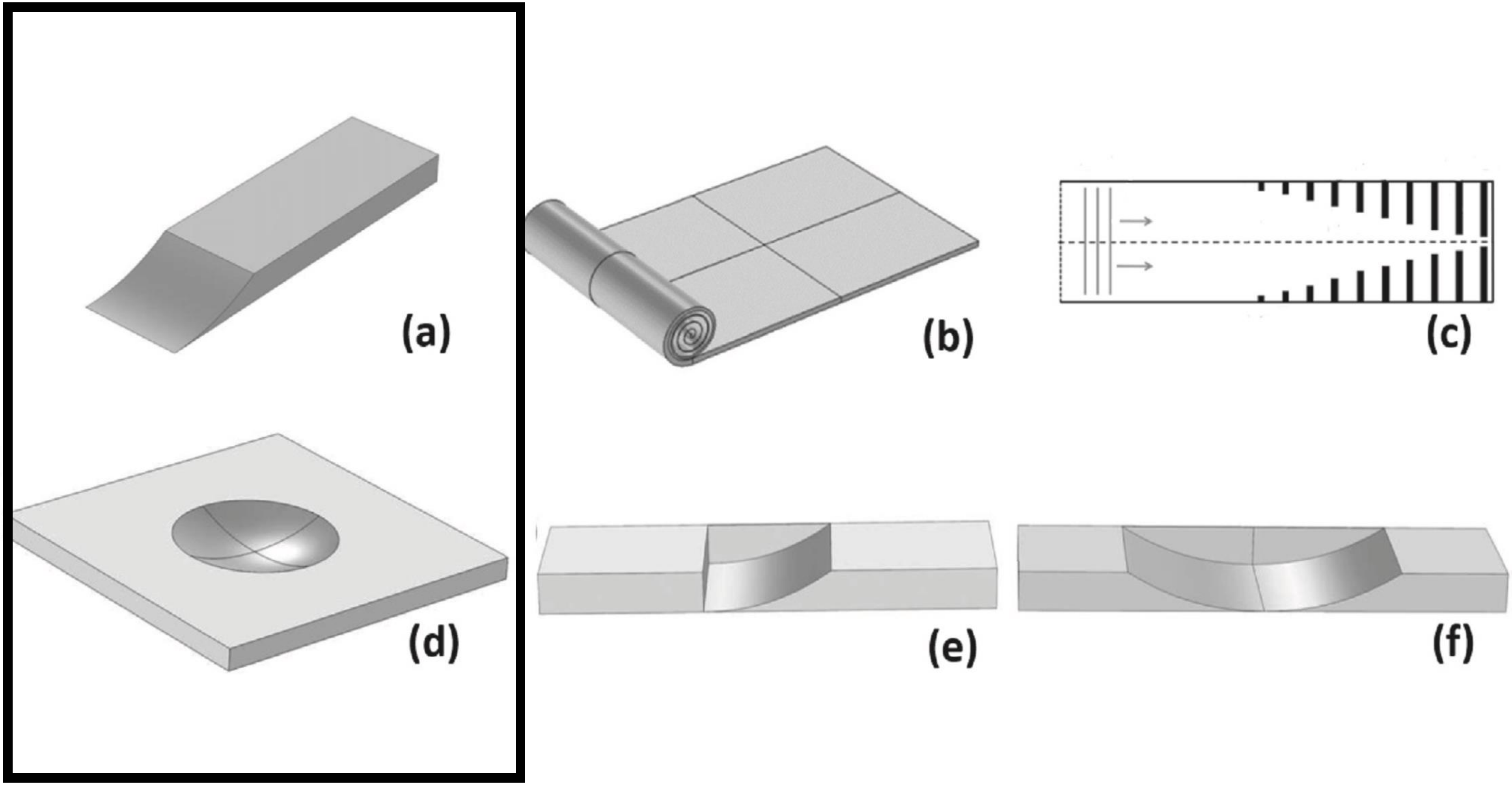

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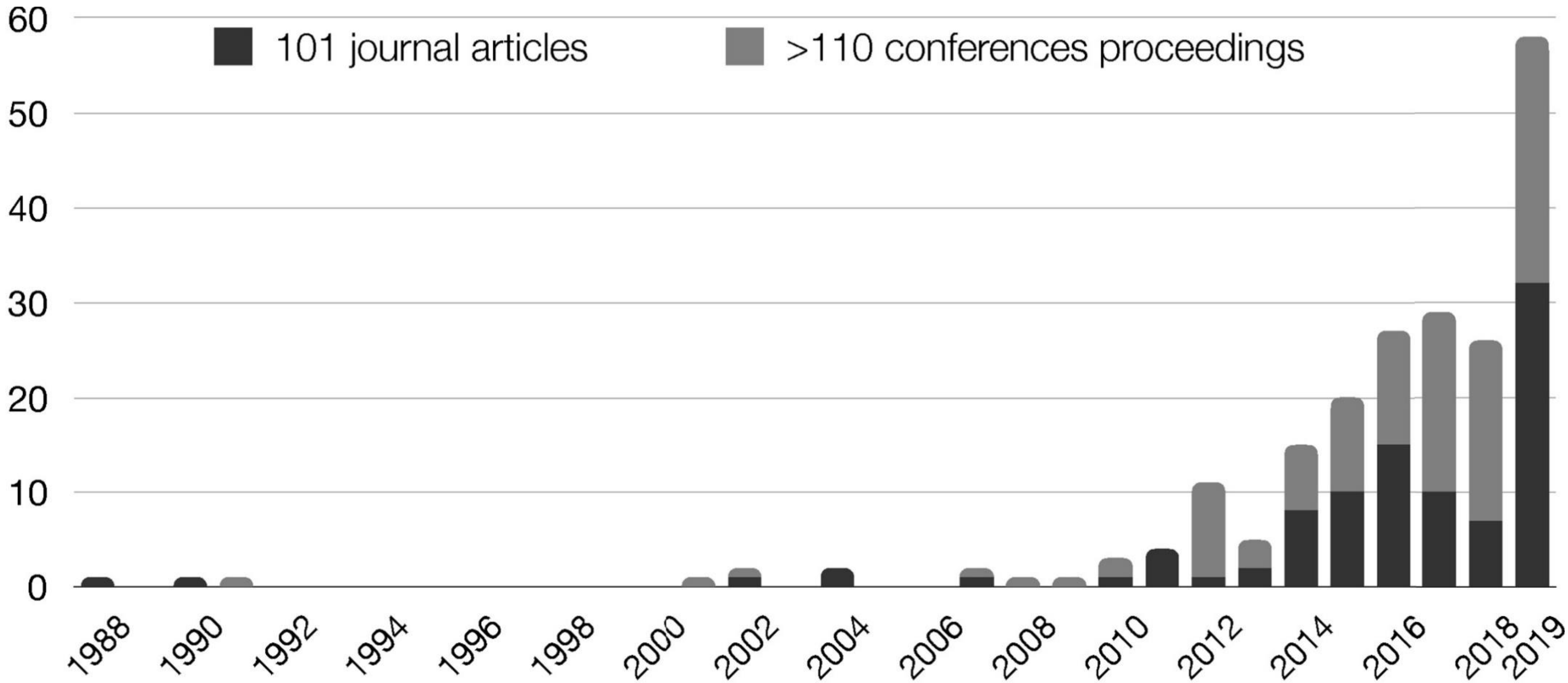
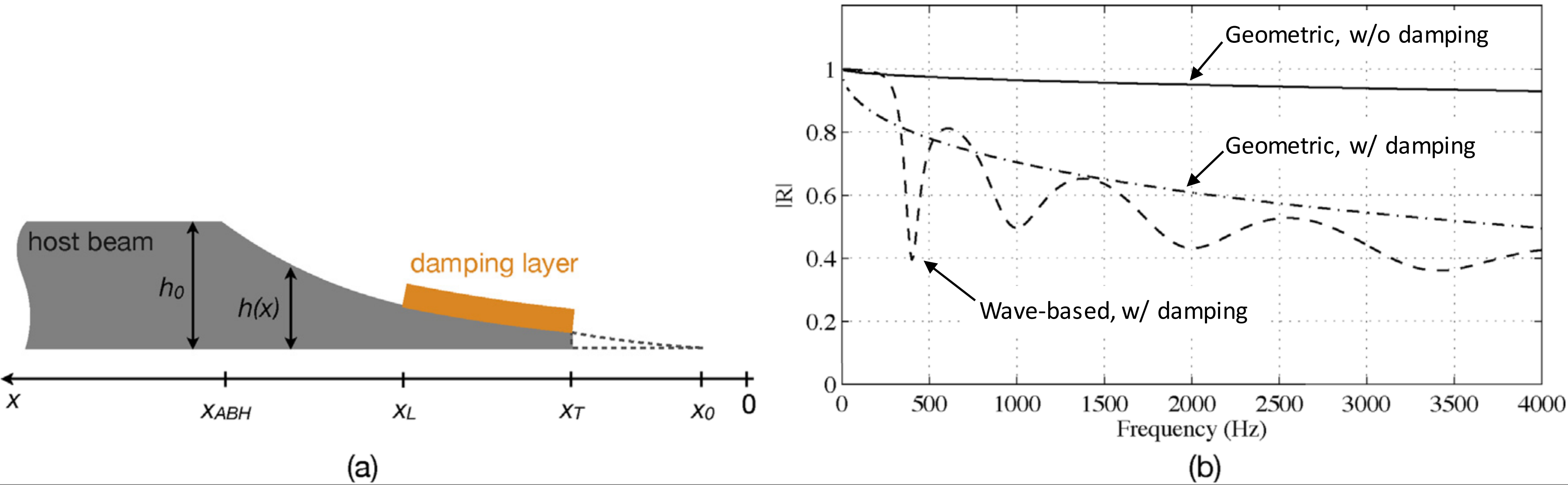


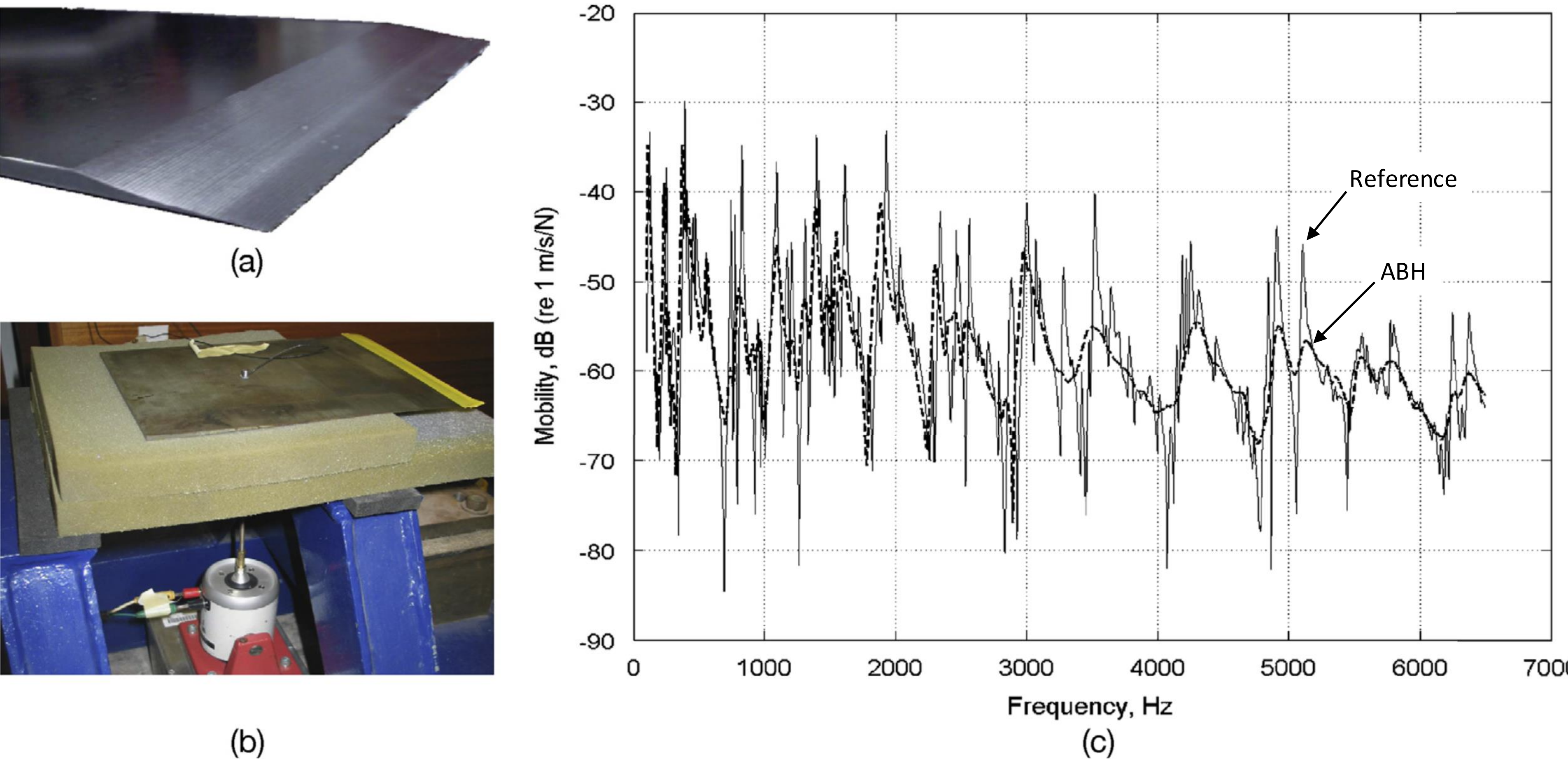
J. Sound Vib. 476, 115316 (2020)

Reflection of imperfect ABH



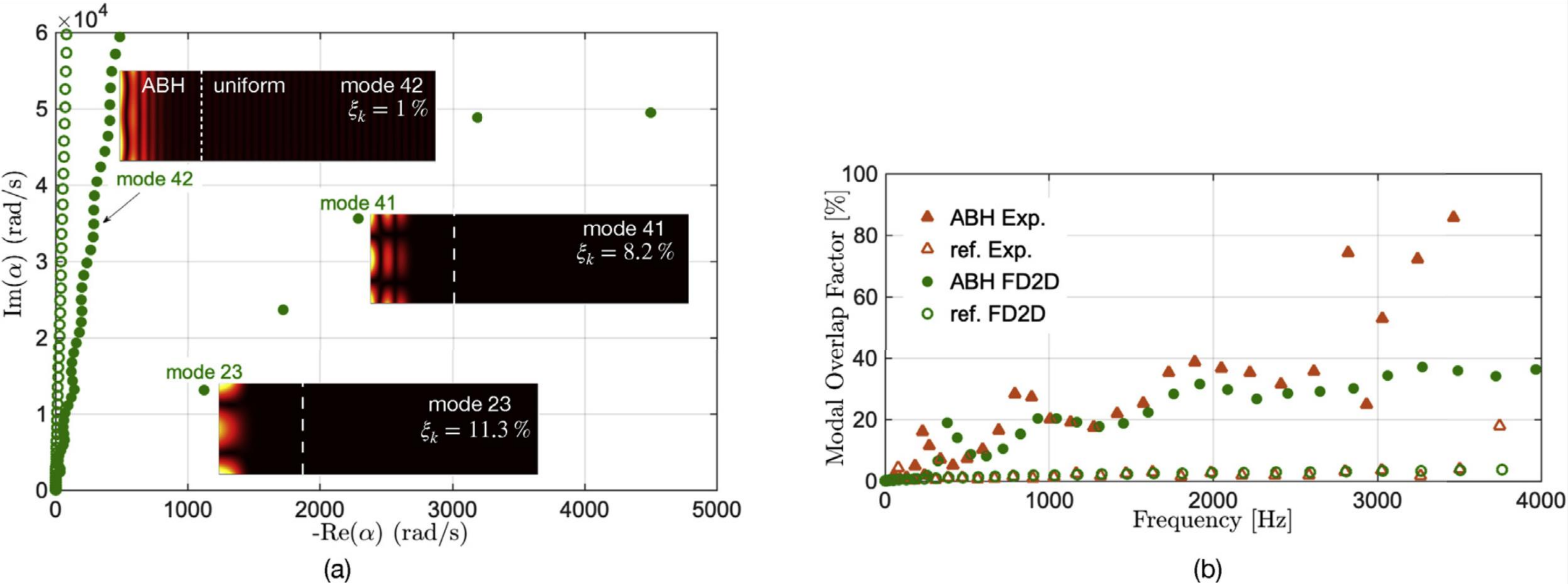
J. Sound Vib. 476, 115316 (2020)

First experiments



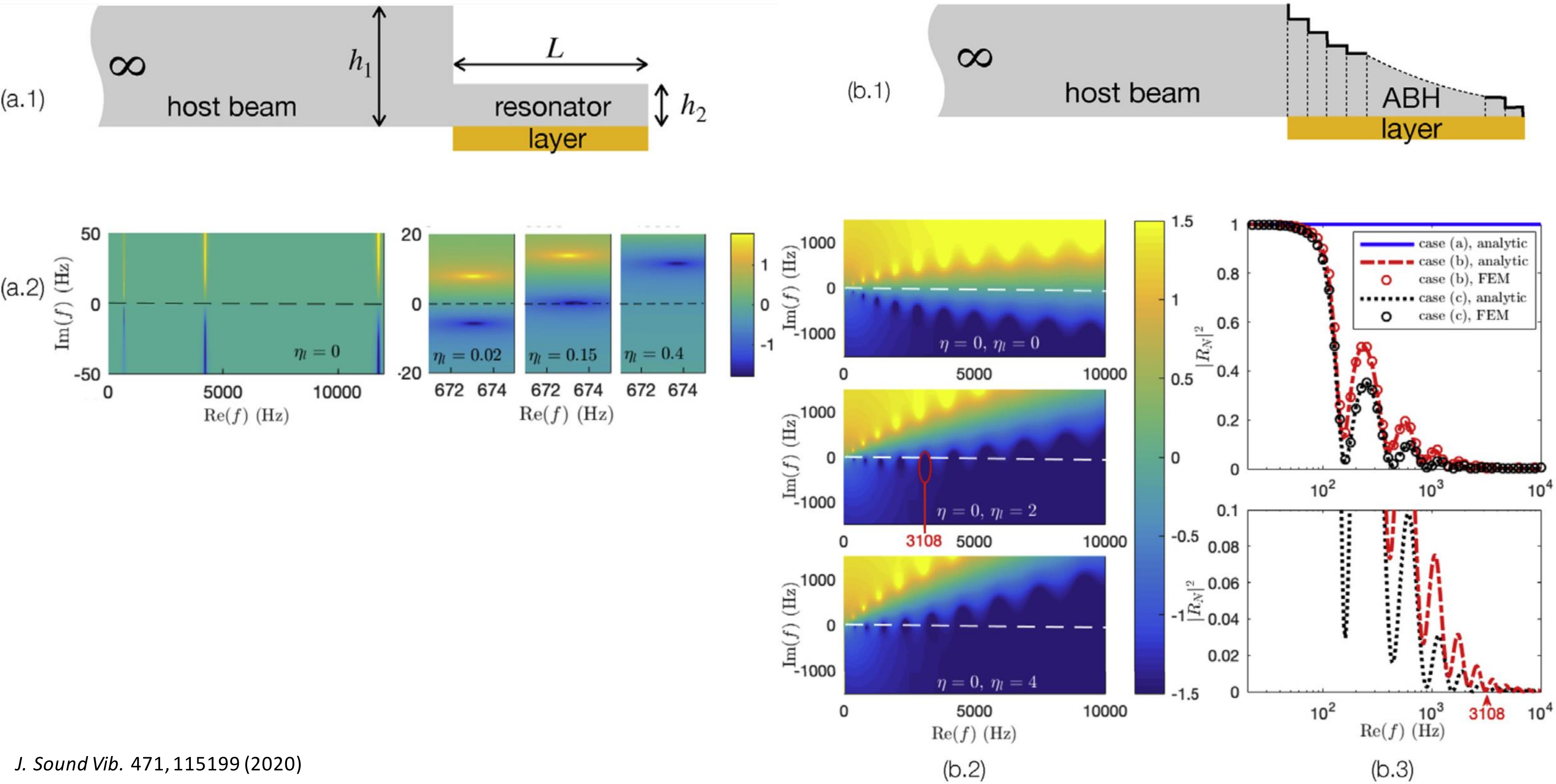
J. Sound Vib. 300(12), 43 (2007)

Modal density and loss factor



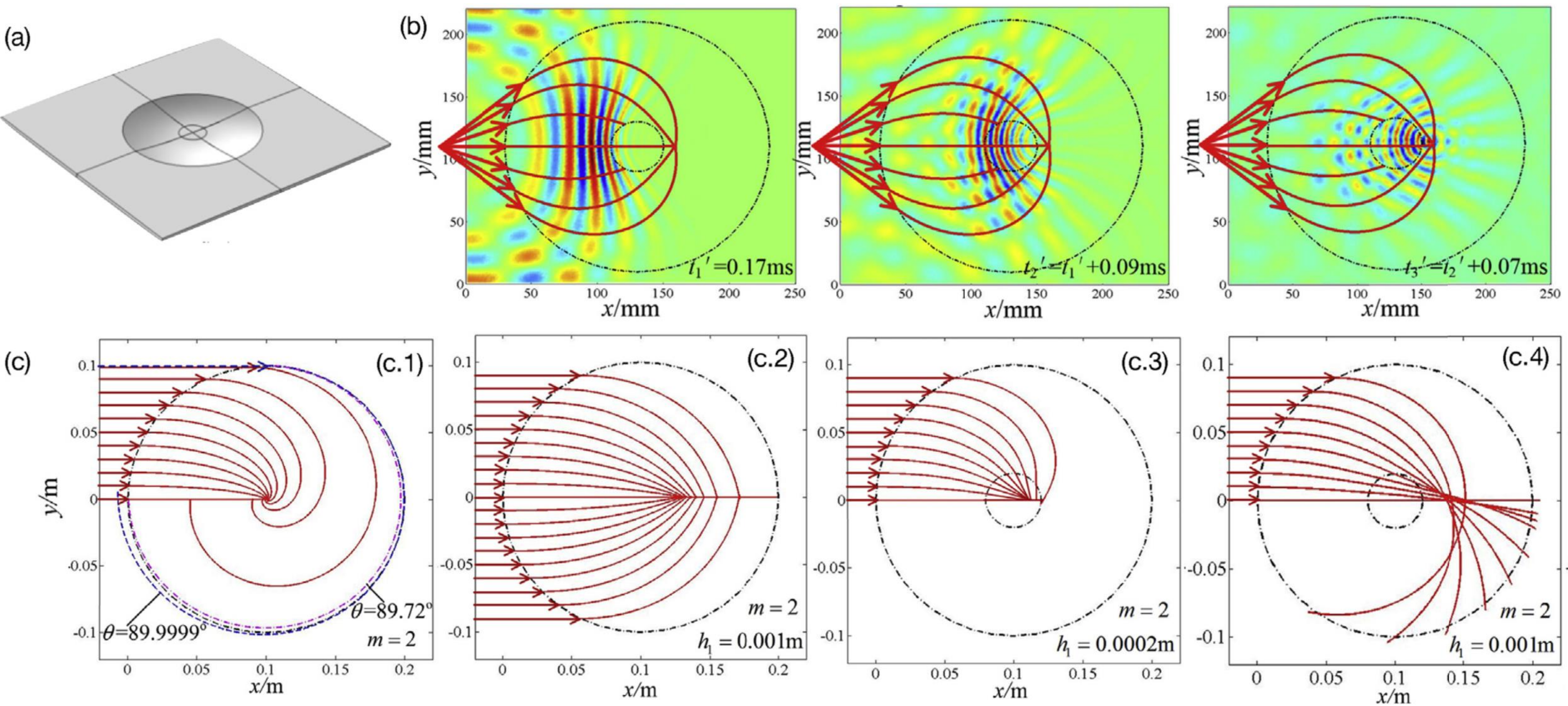
J. Sound Vib. 333(12), 2475 (2014)

Critical coupling



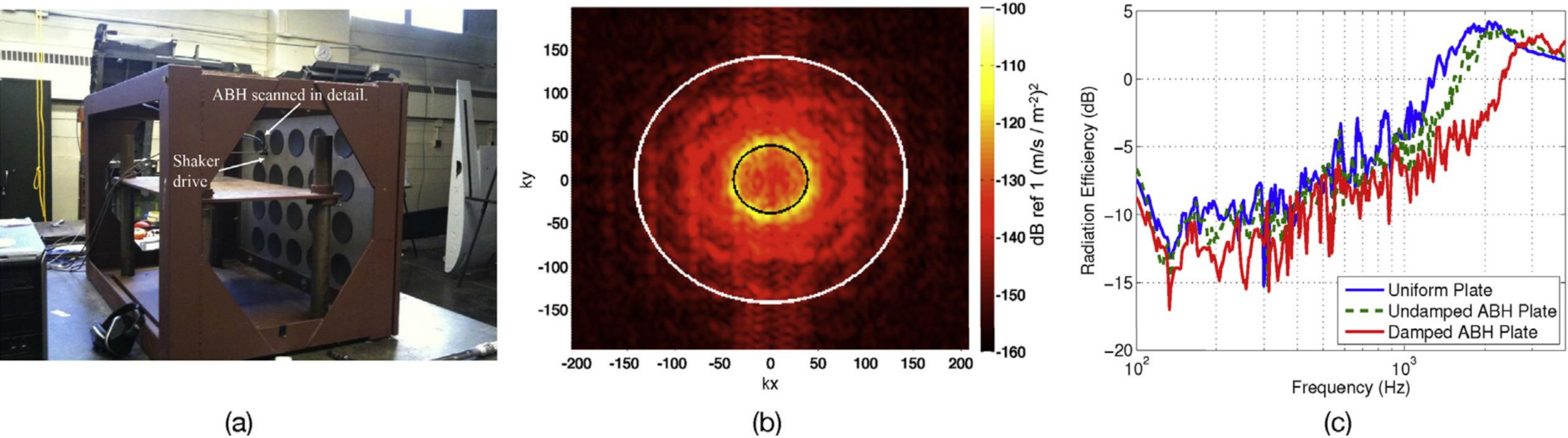
J. Sound Vib. 471, 115199 (2020)

Energy localization



J. Sound Vib. 417, 216 (2018)

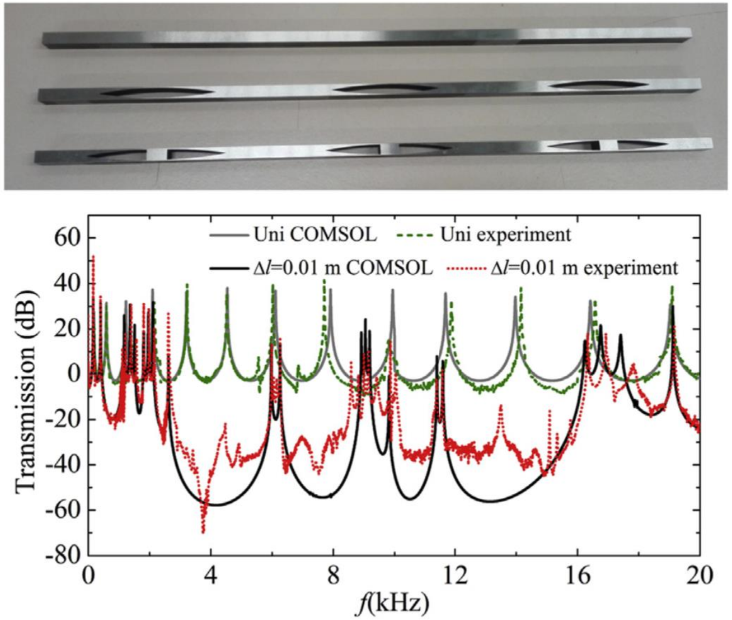
Sound radiation



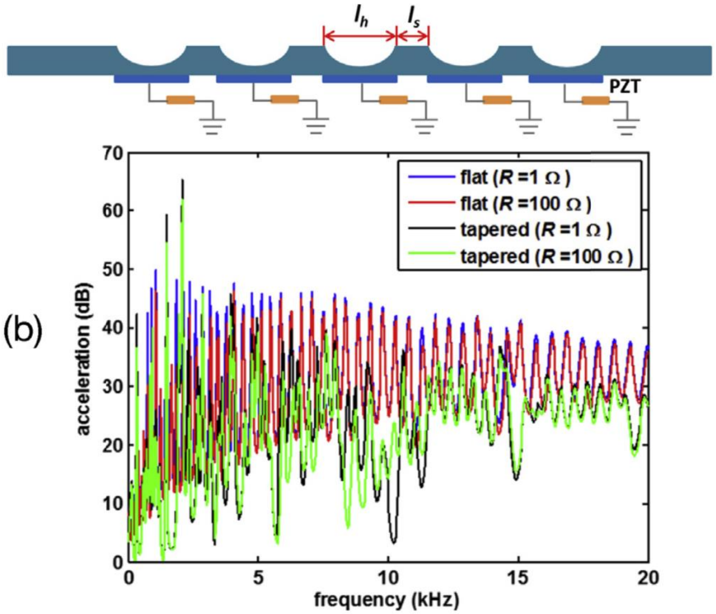
J. Vibr. Acoust. Trans. ASME 138(6), 061002 (2016)

Metamaterials

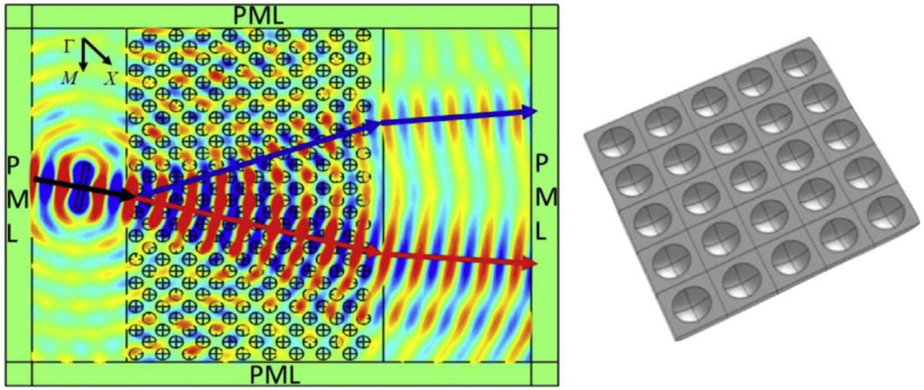
(a)



(b)

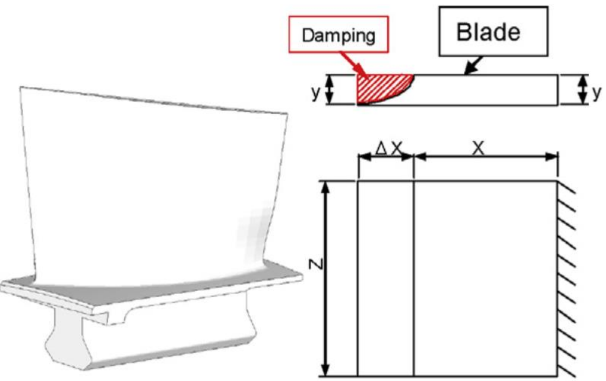


(c)

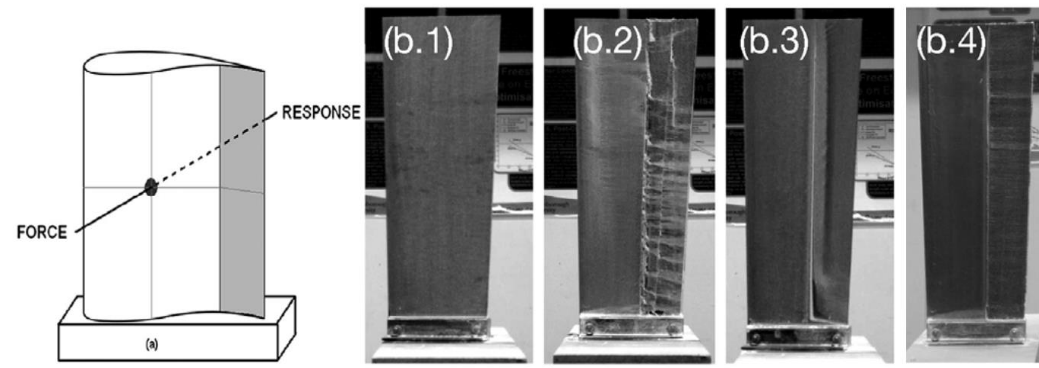


(a) *J. Acoust. Soc. Am.* 142(5), 2802 (2017)
(b) *J. Vibr. Acoust. Trans. ASME* 138(4), 041002 (2016)
(c) *Int. J. Soc. Netw. Min.* 6(1), 1 (2015)

Applications



(a)



(b)

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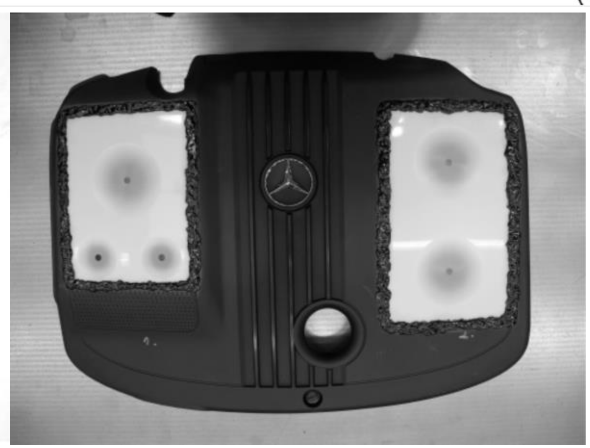

(d)

(a) *J. Syst. Design Dyn.* 5(5), 1176 (2011)
(b) *Appl. Acoust.* 76, 359 (2014)
(c)-(d) *Proc. 44th Int. Con. Exp. Noise Control Eng.*, 6570 (2015)

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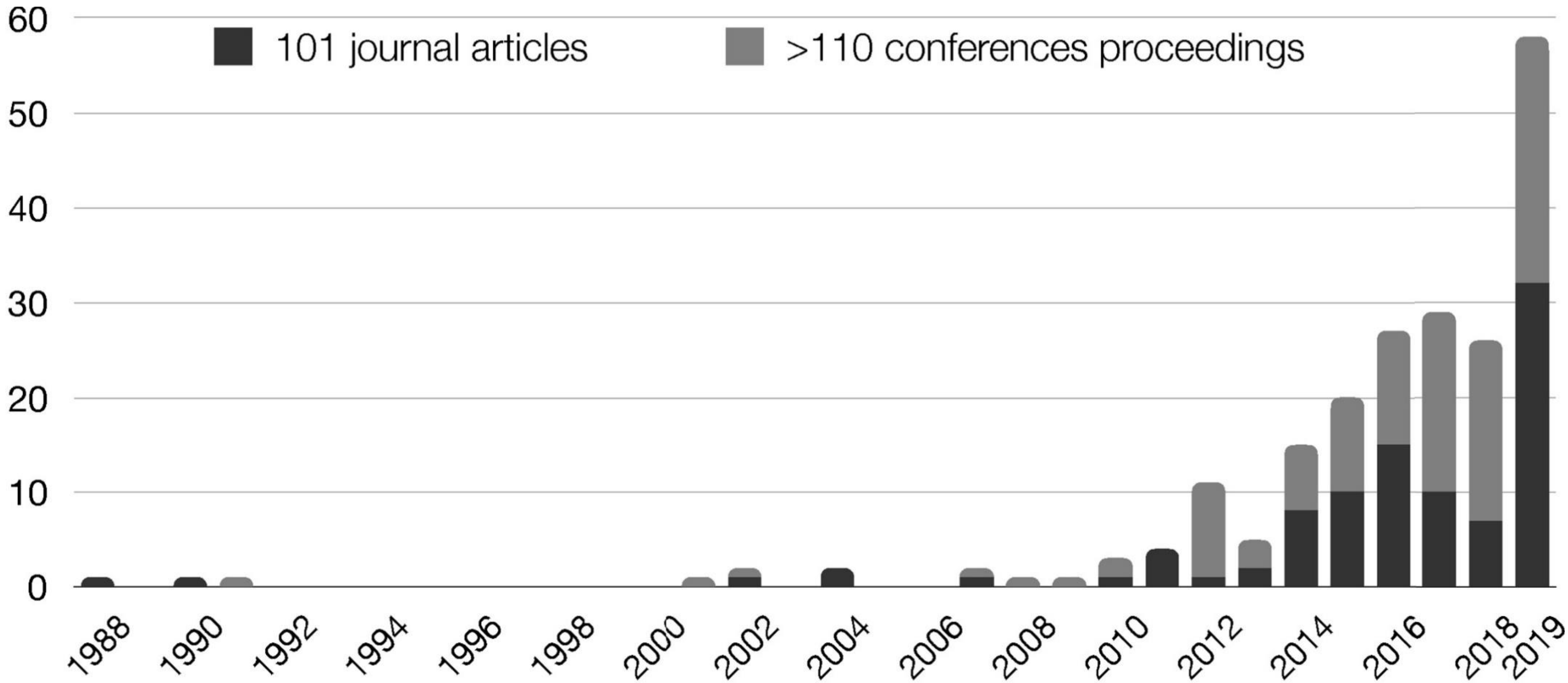

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- 1 before, 1 from, and 1 during the work of this dissertation
- Historically, no clear understanding exists of what constitutes optimal ABH design in practical, finite structures
- By applying formal optimization to several ABH applications and analyzing the results, one can gain a more holistic understanding of the inherent trade-offs in ABH design

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- ABH theory only prescribes the thickness variation, not distribution about the central plane
- Historically, the ABH was modelled/depicted as symmetric about the central plane
- For manufacturing reasons, ABHs are usually formed nonsymmetrically by removing material from one side

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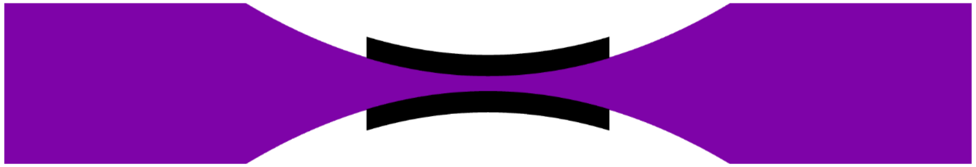
- The choice of ABH ‘style’ is rarely mentioned explicitly in 1D analyses
- The two behave similarly at low frequencies, but not at higher frequencies
- Bowyer and Krylov (2014) and Zhou et al. (2017) proposed a double-layered ABH style that promises unique benefits

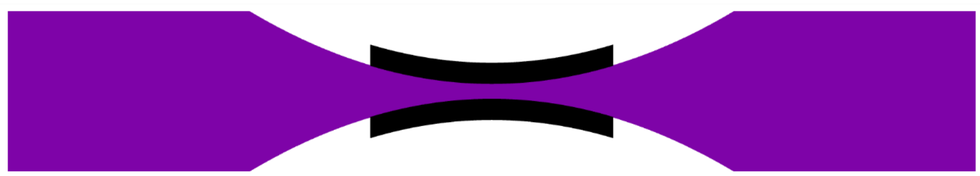
Three styles

Three styles



Standard Symmetric

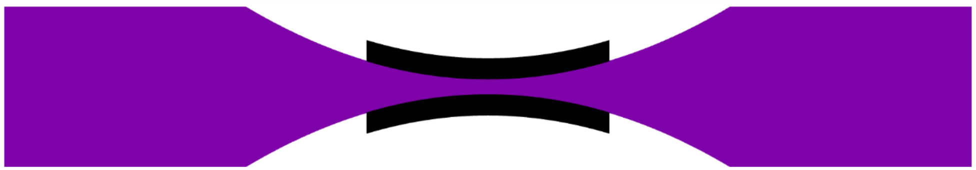
Three styles



Standard Symmetric

Standard Nonsymmetric

Three styles



Standard Symmetric

Standard Nonsymmetric

Double-leaf

Problem description

- 1D ABH vibration absorber embedded in a simply supported plate

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Problem description

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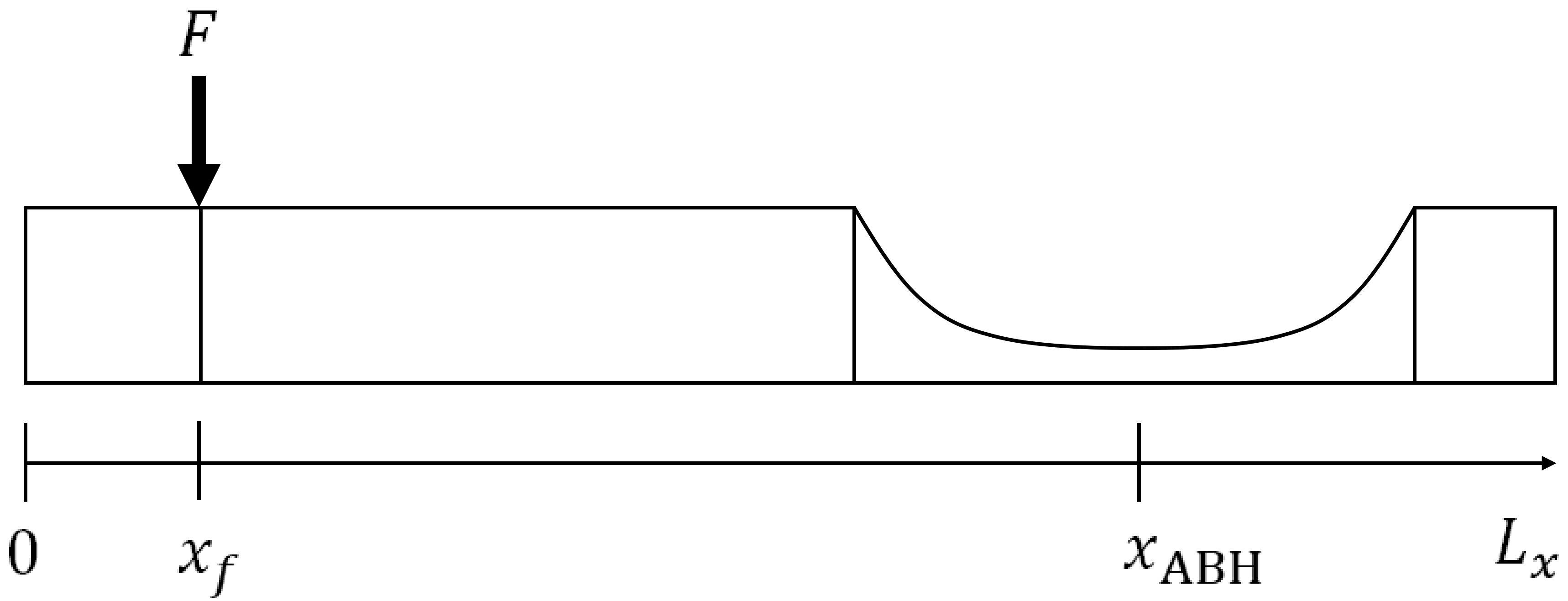
- Excited by a harmonic point force
- ABH taper shape is restricted to $h(x) = \varepsilon x^m + h_0$ but otherwise allowed to vary continuously
- Position of ABH and amount of added damping may also vary continuously
- Objective is to simultaneously minimize the total mass and the spatially-averaged squared velocity response of the entire length

Problem description

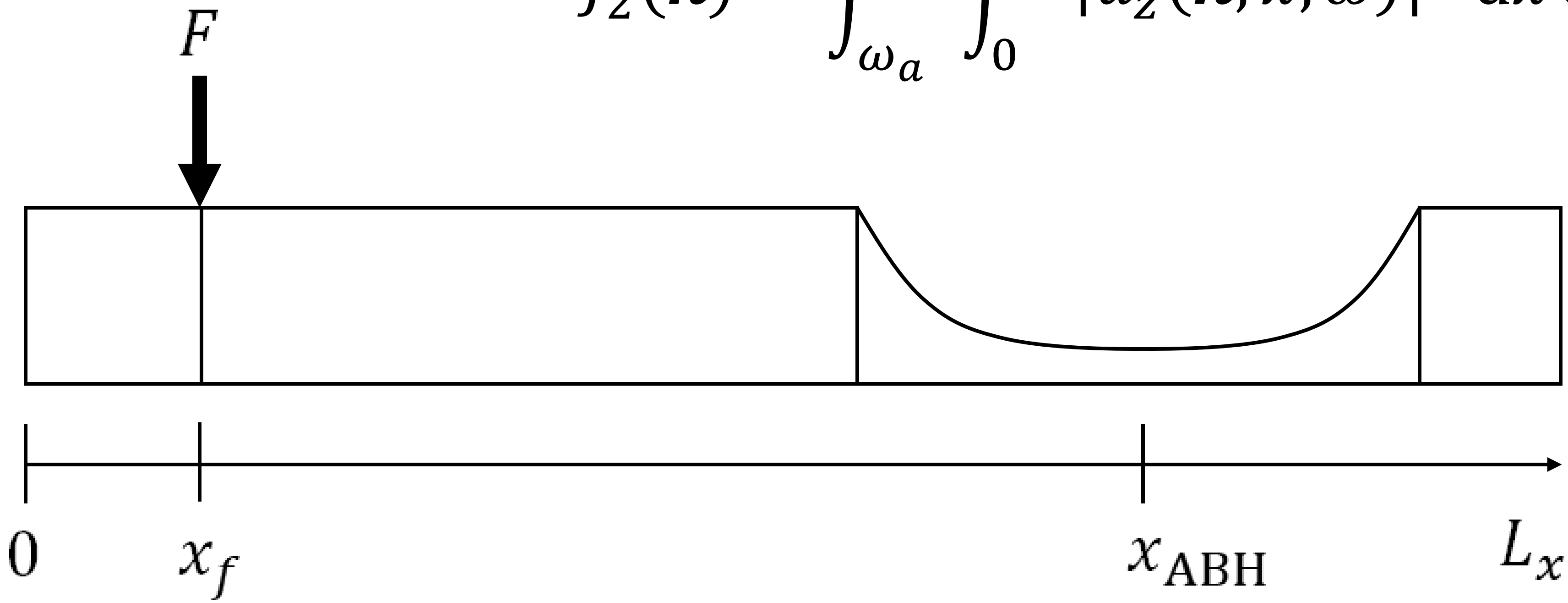
Design variables: $$\boldsymbol{h} = [L_{\text{ABH}} \quad h_0 \quad m \quad x_{\text{ABH}} \quad P_d]^{\text{T}}$$



Problem description

Objective functions: $J_1(\boldsymbol{h}) = M_{tot}$

$$J_2(\boldsymbol{h}) = \int_{\omega_a}^{\omega_b} \int_0^{L_x} |\dot{u}_z(\boldsymbol{h}, x, \omega)|^2 \, \mathrm{d}x \, \mathrm{d}\omega$$



Problem description

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Constraints:

1) Plane strain elastodynamic eq.
2) $\boldsymbol{h} \in \mathcal{H}$
3) Pinned BCs

Numerical model

2D FEM using NASTRAN

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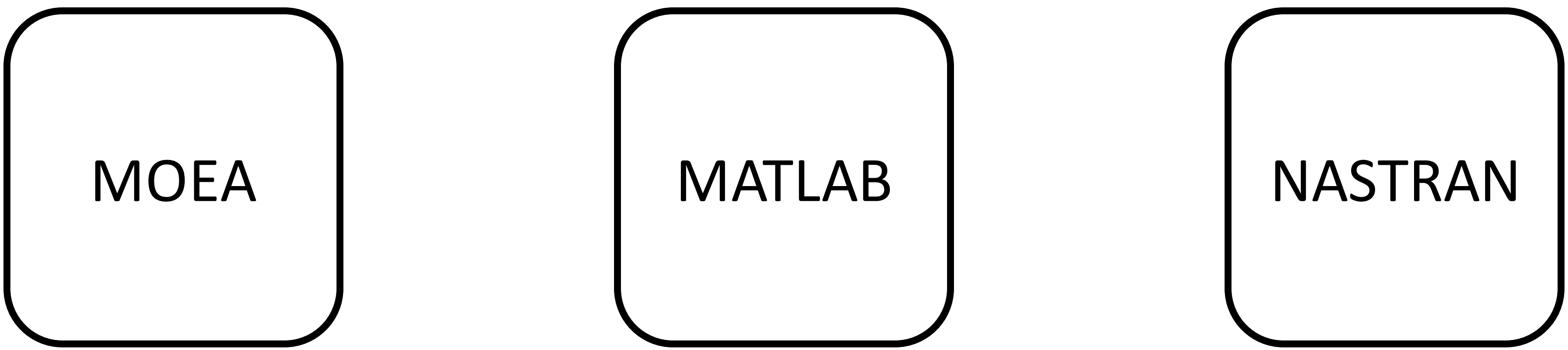

Numerical model

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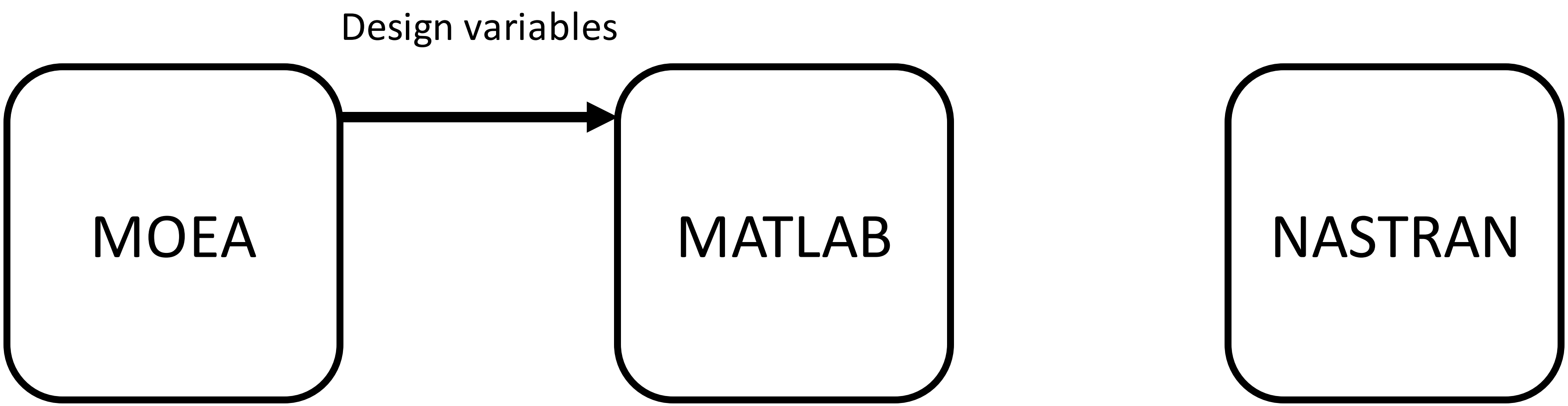

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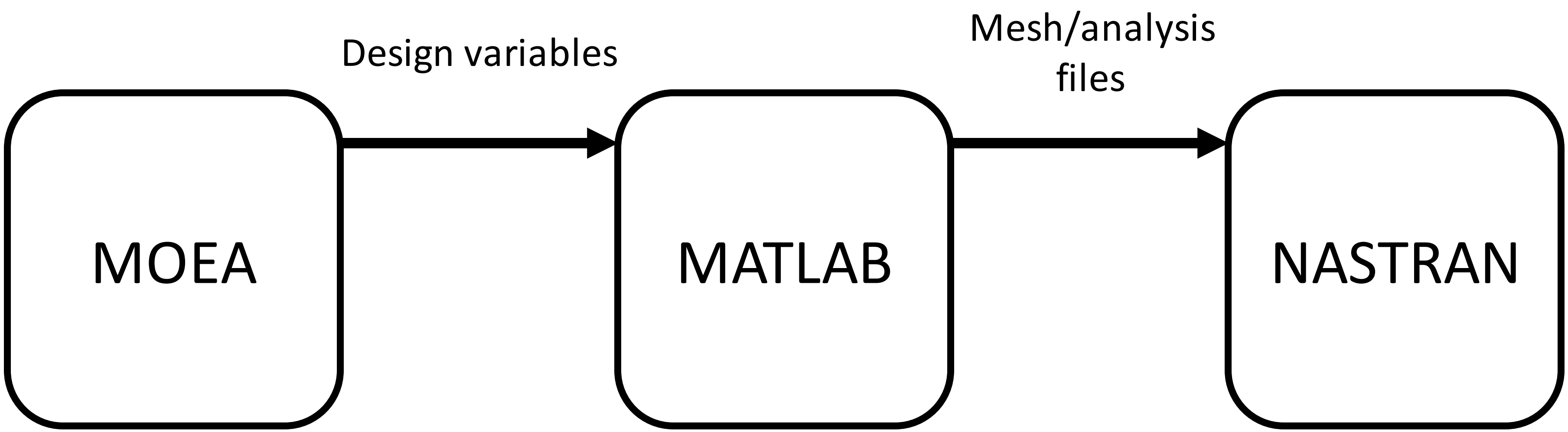

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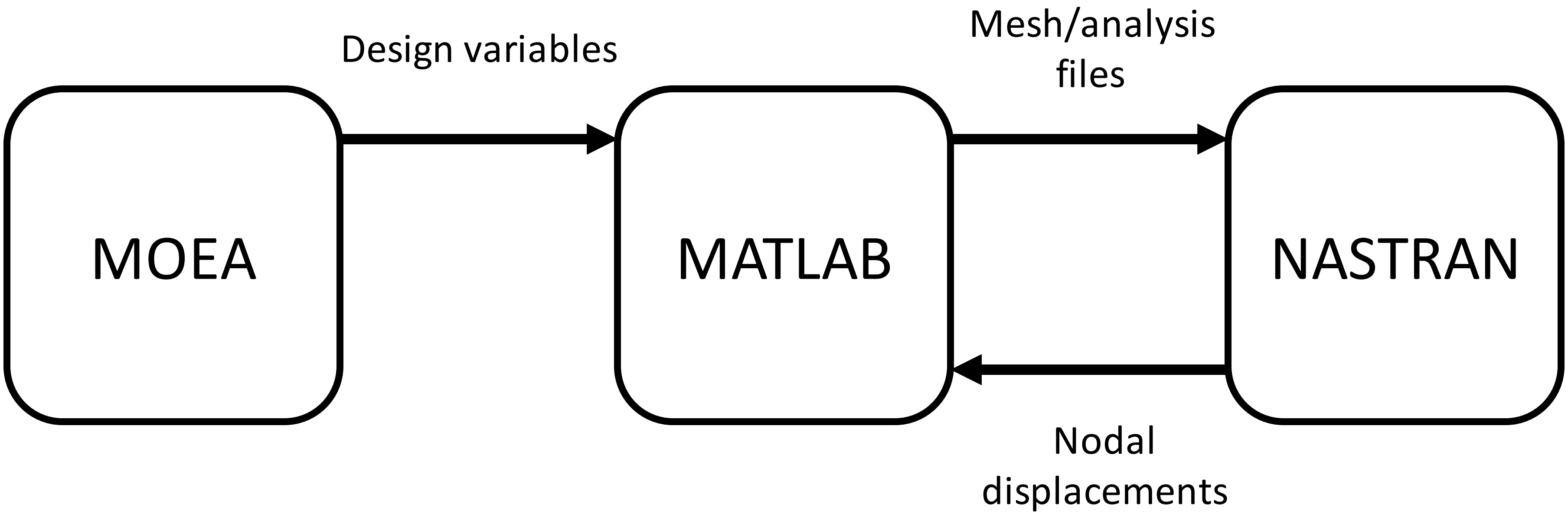

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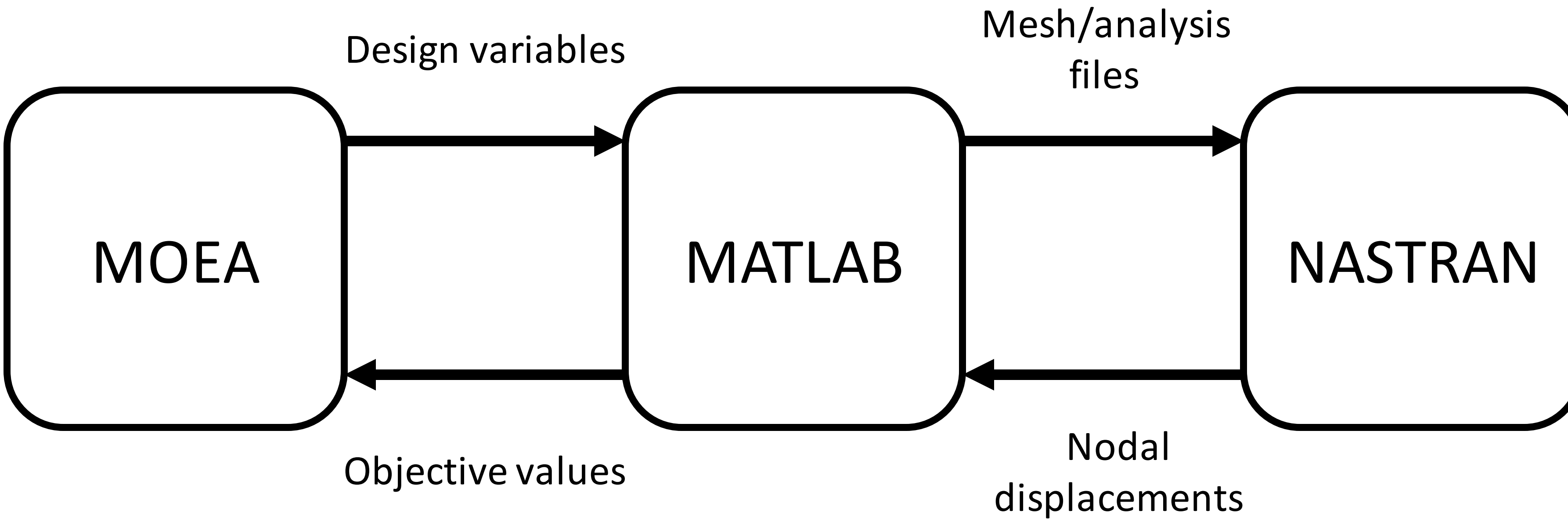
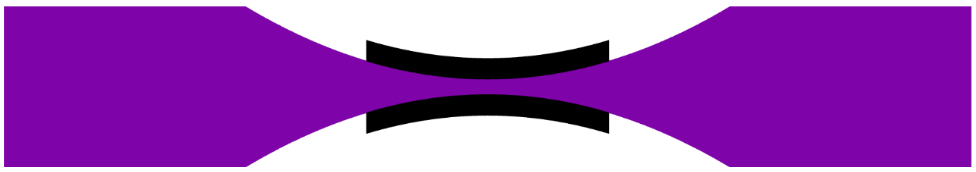
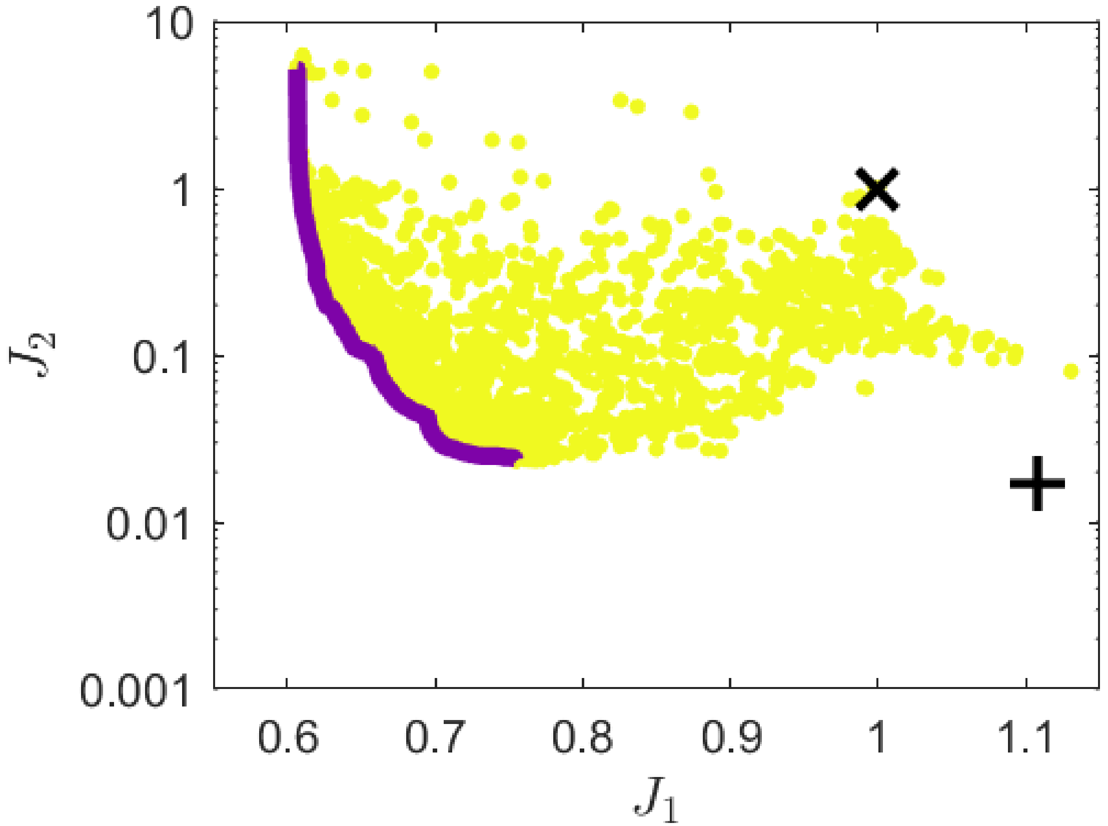


Results - Pareto fronts

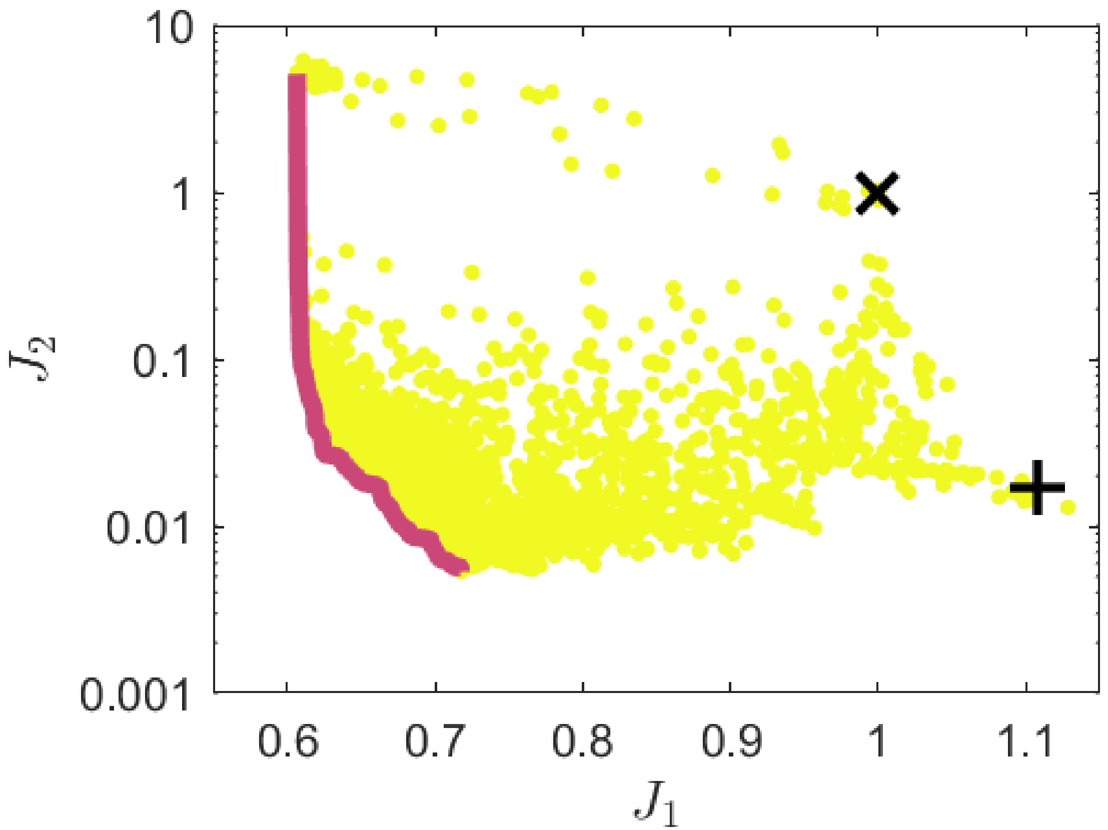


Standard Symmetric

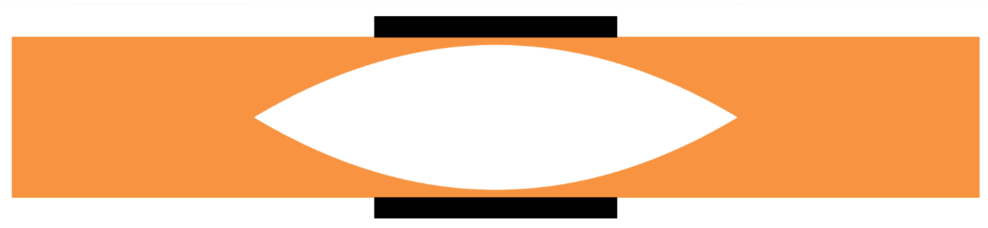


Results - Pareto fronts

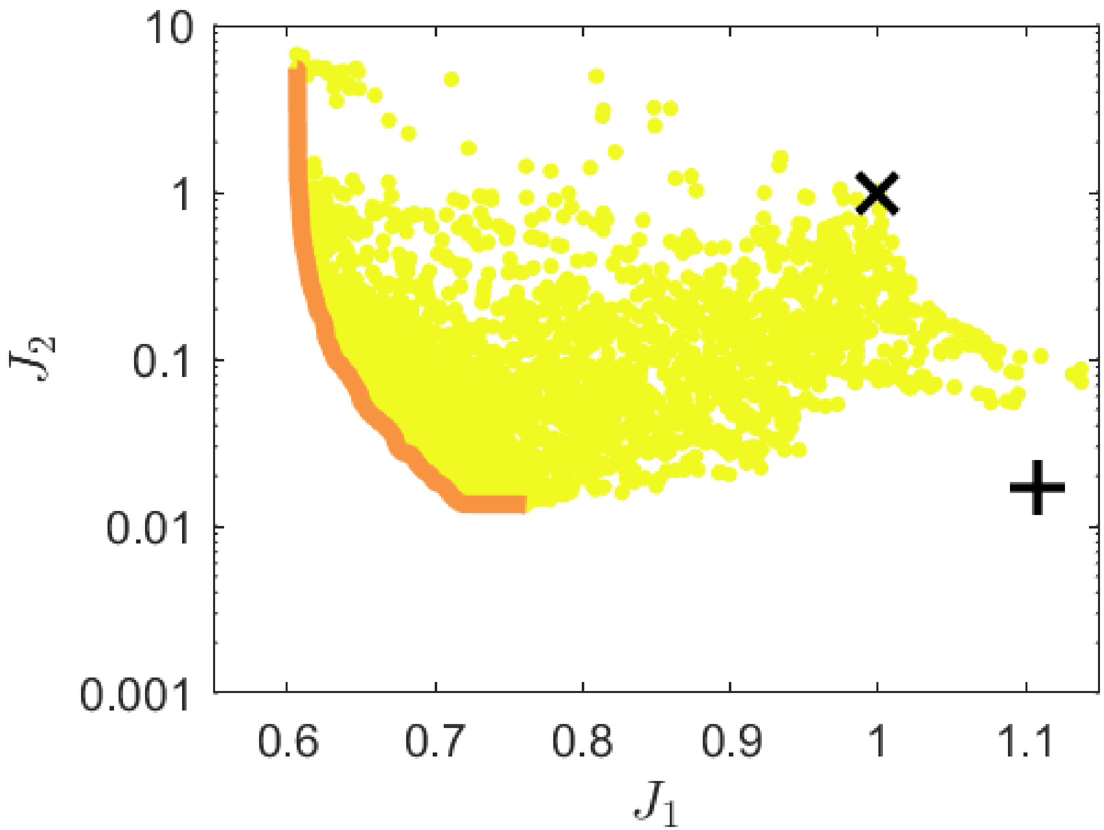
Standard Nonsymmetric



Results - Pareto fronts



Double-leaf



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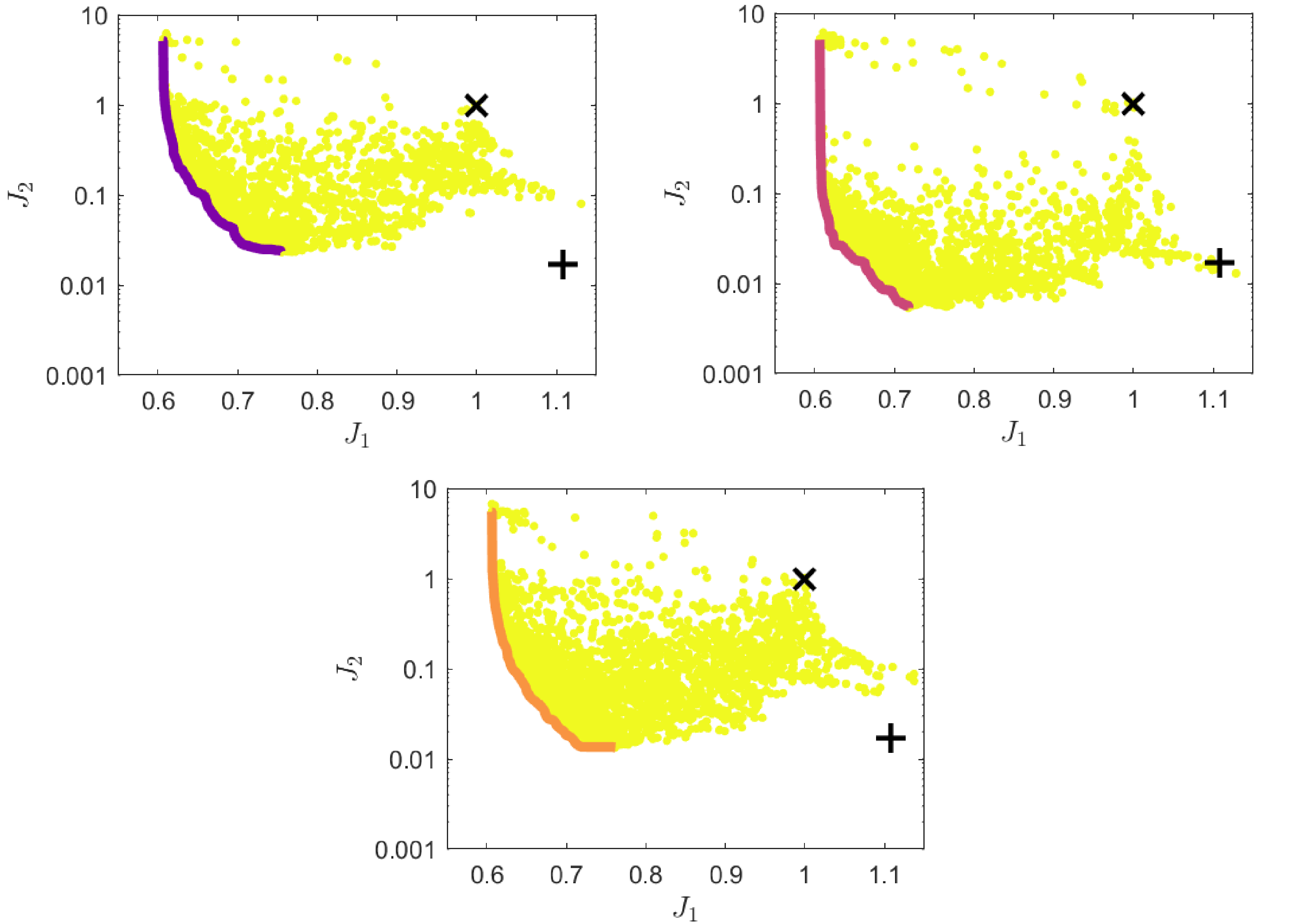
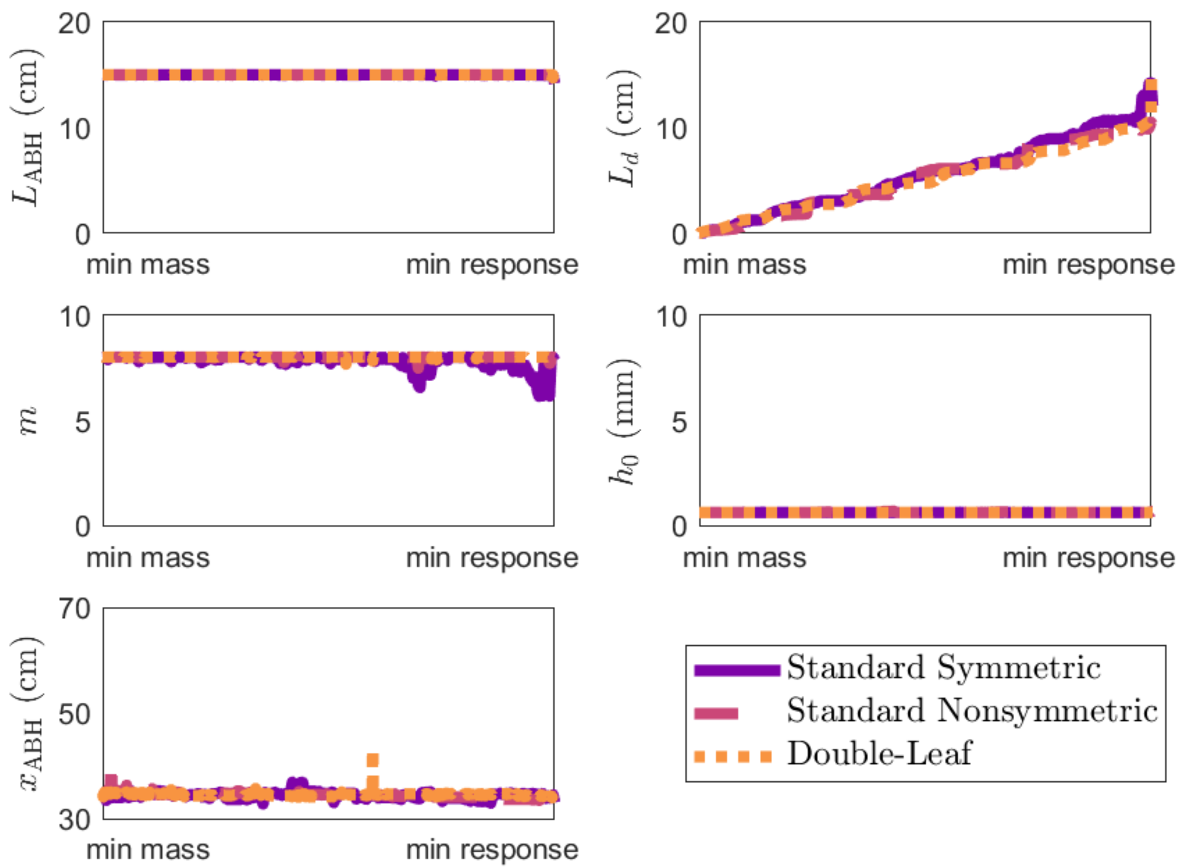
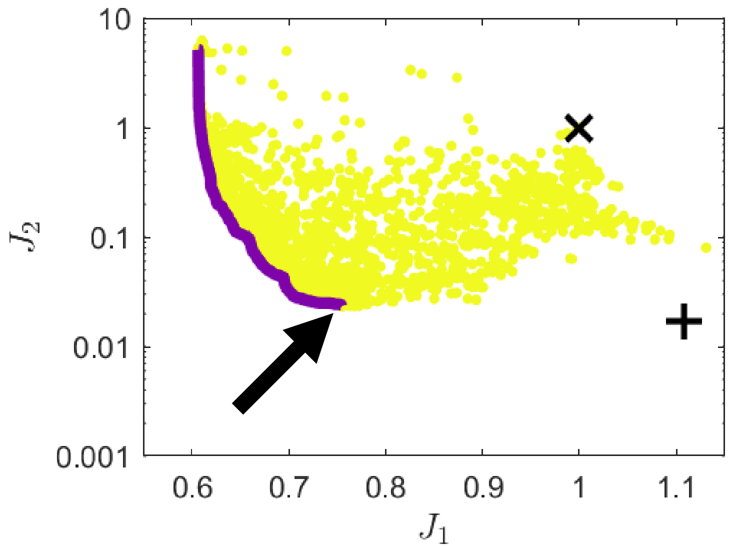
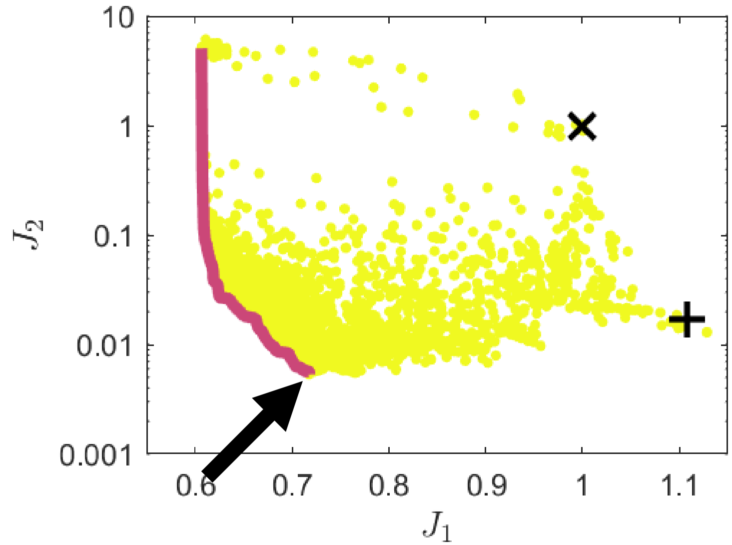


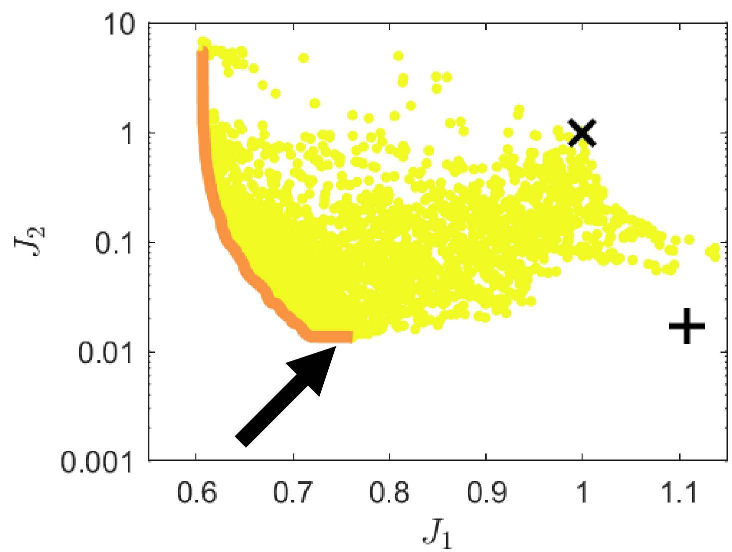
Results - Variable trends



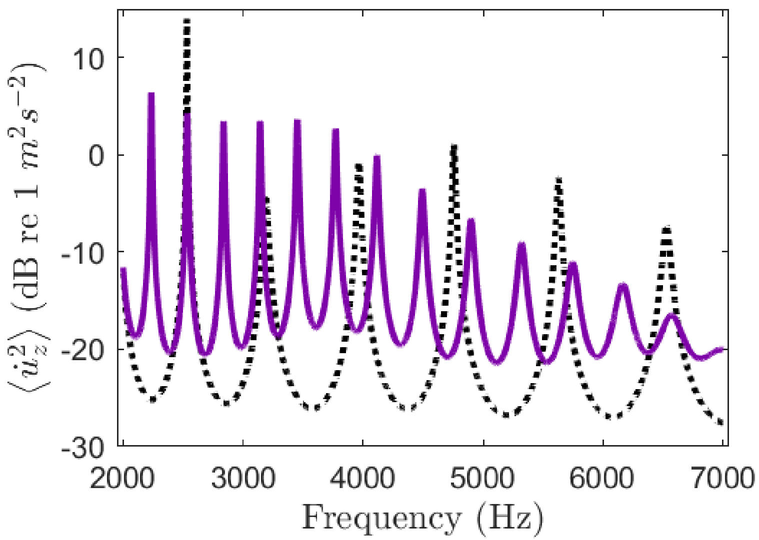
Results - Threshold designs

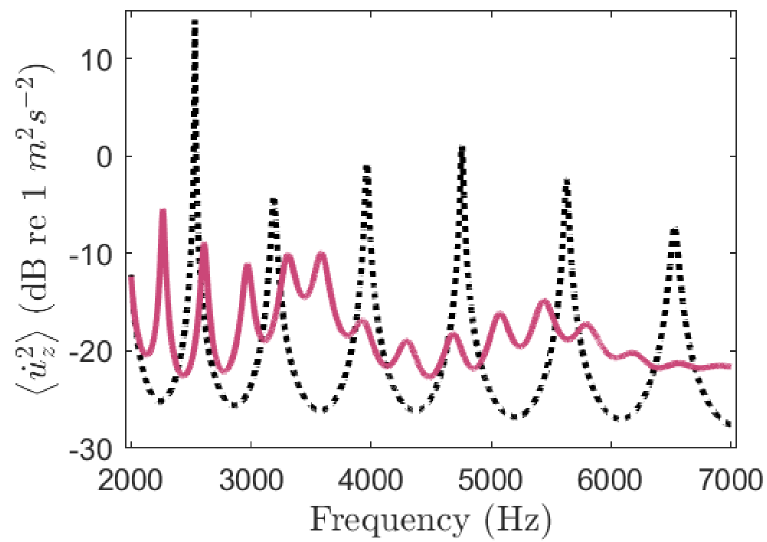


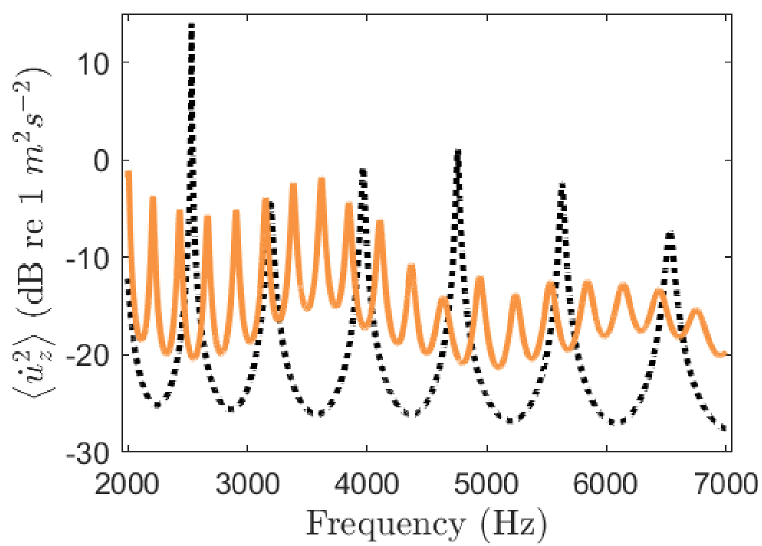




Results - Velocity spectra

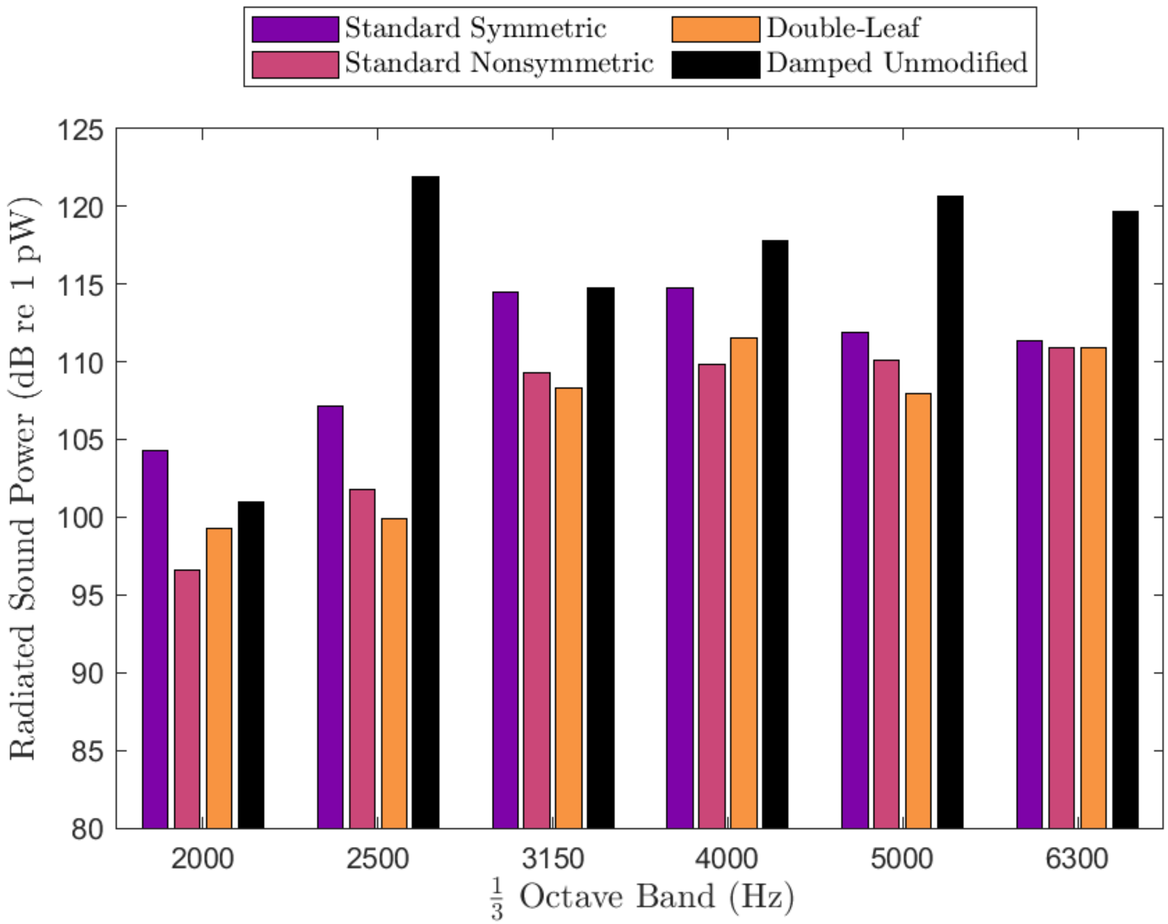






Dashed curve represents an unmodified beam with the maximum damping allowed

Results - Radiated sound



Sound power calculated by the Rayleigh integral for $k \leq k_{\mathrm{air}}$

Q: To what extent is modelling the second dimension important?

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A: Modelling the second dimension is critical to accurately modelling the ABH effect

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  - *J. Sound Vib.* 470, 115164 (2020)

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  - *J. Sound Vib.* 470, 115164 (2020)
- Modelling the second dimension is critical to accurately modelling the ABH effect
- Precise choice of style may vary by application, but Standard Nonsymmetric is superior for vibration reduction
- Damping design can be critical for effective ABH performance
  - Multiple damping studies followed these results

Structure of the Talk

- Background
- Review of ABH research
- ABH optimization study
- Concluding remarks

Summary

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Summary

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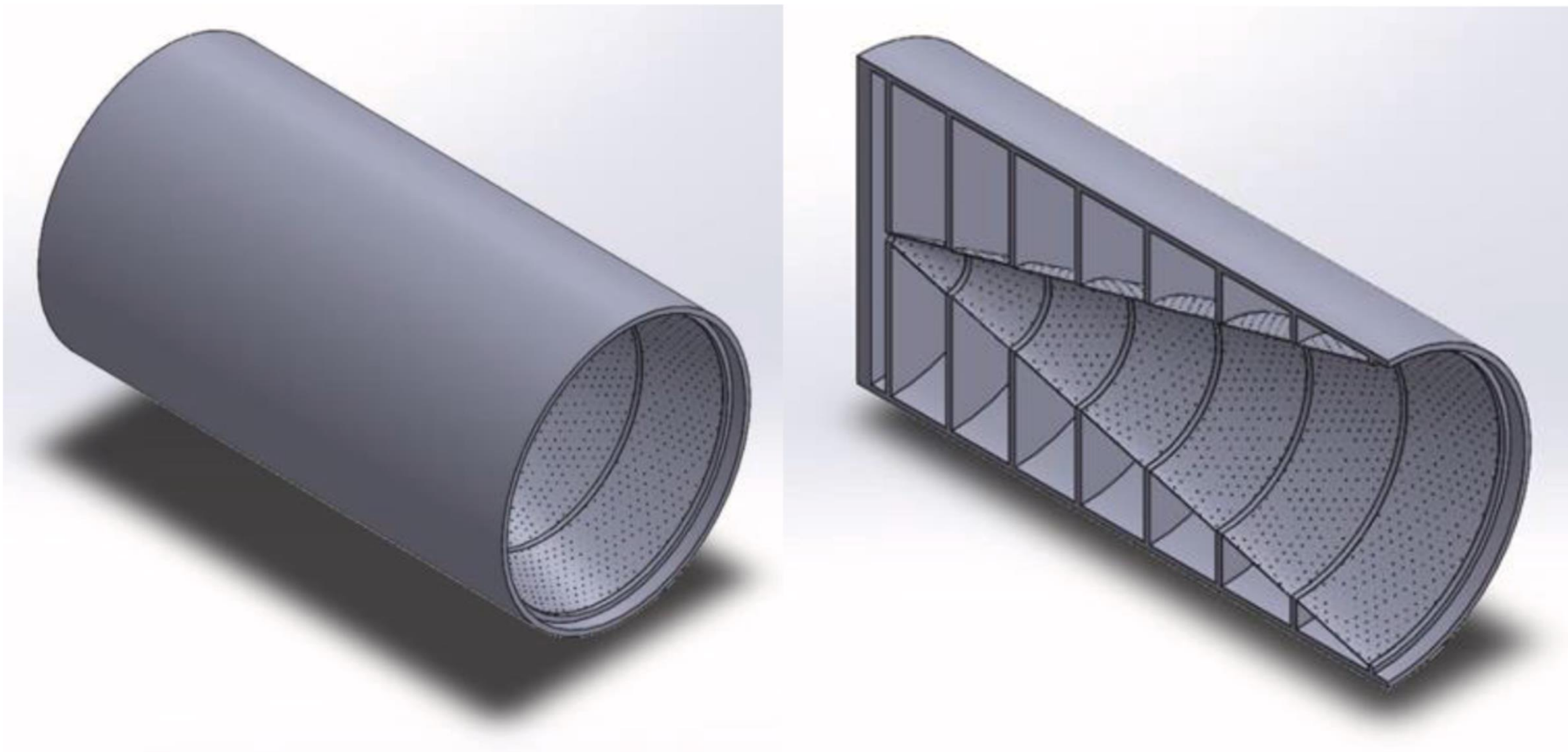
- Results in smaller wavelength and larger amplitude (energy localization) and slower wavespeed (acoustic ‘decoupling’)
- Great amount of flexibility in implementation, but implementation matters
- Damping design also an important factor

Epilogue - Sonic black hole



(a)

(b)

J. Sound Vib. 512, 116401 (2021)

Thank You

Primary references

- A. Pelat, F. Gautier, S. C. Conlon, and F. Semperlotti, "The acoustic black hole: A review of theory and applications," *J. Sound Vib.* 476, 115316 (2020)
  - DOI: 10.1016/j.jsv.2020.115316

- C. Zhao and M. G. Prasad, "Acoustic black holes in structural design for vibration and noise control," *Acoustics* 1(1), 220-251 (2019).
  - DOI: 10.3390/acoustics1010014

Results - Static compliance (axial)

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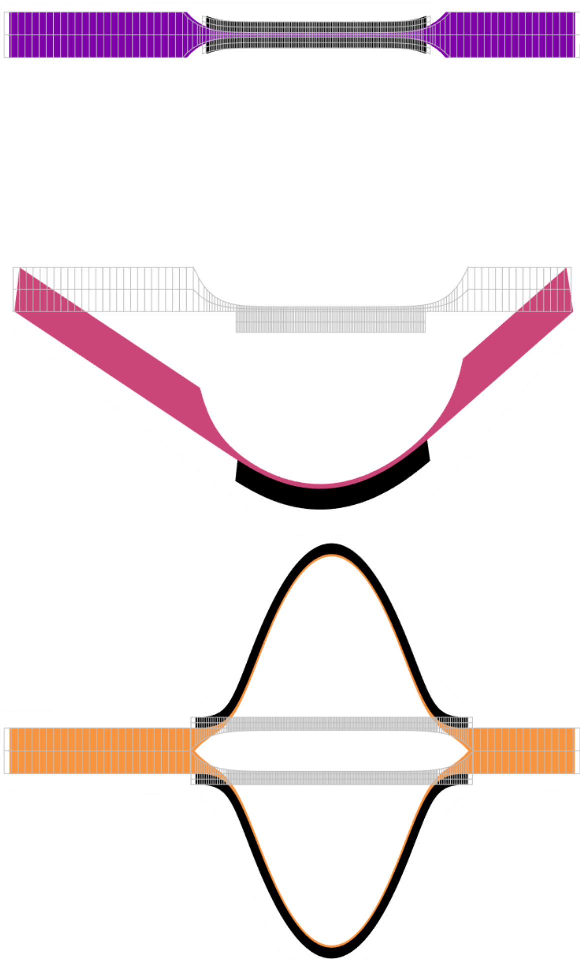

	Linear strain
Standard Symmetric	3.4×10^{-5}
Standard Nonsymmetric	9.4×10^{-2}
Double-leaf	3.2×10^{-5}
Unmodified	8.3×10^{-6}

Results - Static compliance (torque)

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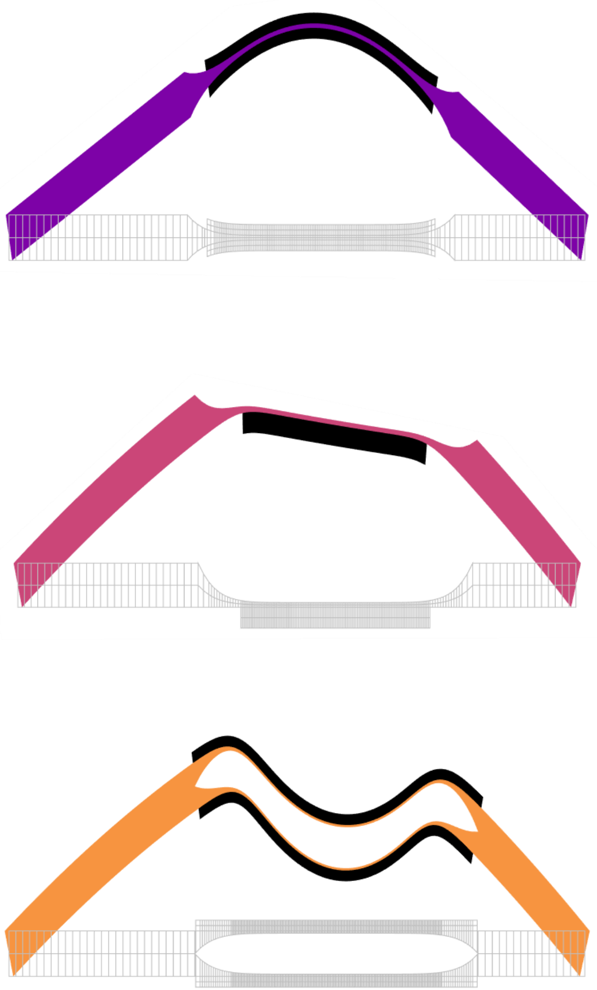

	Linear strain
Standard Symmetric	8.0×10^{-2}
Standard Nonsymmetric	2.2×10^{-5}
Double-leaf	2.3×10^{-5}
Unmodified	4.3×10^{-7}

Results - Summary

	Vibration	Sound	Stiffness
Standard Symmetric			
Standard Non-symmetric	X	X	
Double-leaf		X	X

Borg MOEA

- Multi-objective evolutionary algorithm

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- Draws inspiration from ε-MOEA, ε-NSGA-II, and AMALGAM
- Shown to outperform other MOEAs on benchmark problems
- Used in previous structural optimization; outperformed standard gradient-based method

Borg MOEA - Key features

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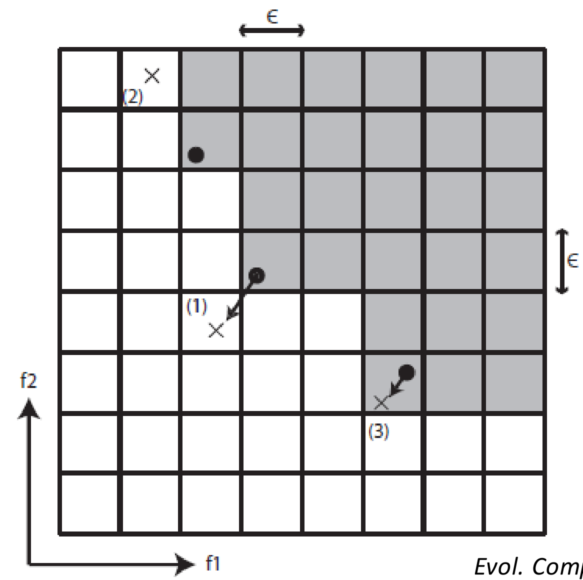
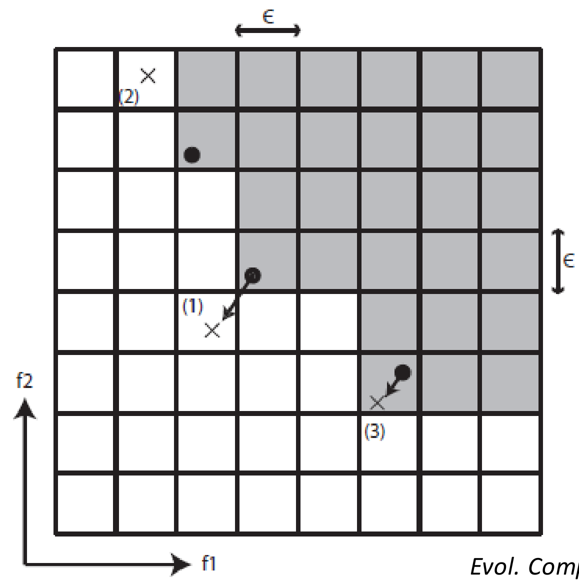
- ε-Progress



Evol. Comp. 21(2), 231-259 (2013)

Borg MOEA - Key features

- ε-Progress
- Automatic restarts



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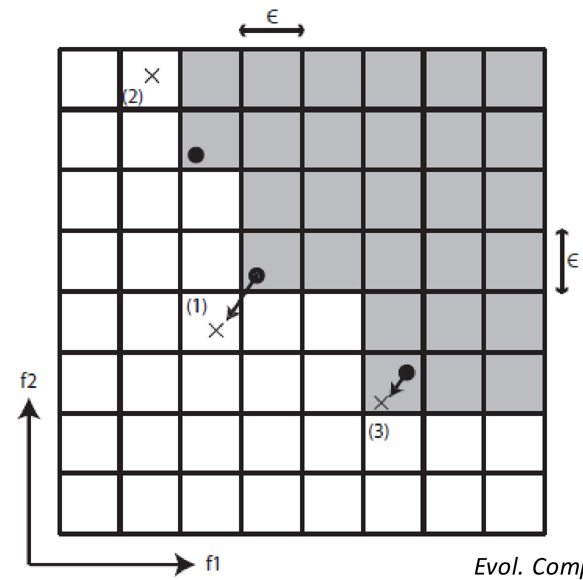

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Borg MOEA - Key features

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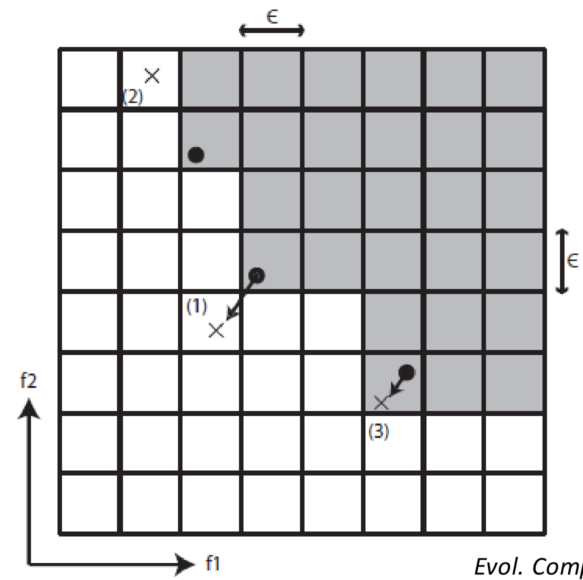
- Adaptive population & tournament resizing
- Multi-operator recombination



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